

Answer on Question #65197, Math / Calculus| for confirmation

Limit Question:

$$\lim_{x \rightarrow \infty} (1 + x^2)^{e^{-x}} = \lim_{x \rightarrow \infty} e^{\ln((1+x^2)^{e^{-x}})} = \lim_{x \rightarrow \infty} e^{\frac{\ln(1+x^2)}{e^x}} = e^{\lim_{x \rightarrow \infty} \frac{\ln(1+x^2)}{e^x}} = e^0 = 1.$$

Use of L'Hôpital's Rule (Infinity over Infinity):

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{\ln(1+x^2)}{e^x} &= \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow \infty} \frac{(\ln(1+x^2))'}{(e^x)'} = \lim_{x \rightarrow \infty} \frac{2x}{1+x^2} = \lim_{x \rightarrow \infty} \frac{2x}{(1+x^2)e^x} = \\ &= \lim_{x \rightarrow \infty} \frac{2}{\left(\frac{1}{x} + x\right)e^x} = 0 \end{aligned}$$

Answer:

$$\lim_{x \rightarrow \infty} (1 + x^2)^{e^{-x}} = 1.$$