Answer on Question #65190 – Math – Analytic Geometry

Question

Find the equation of the sphere which passes through the points $(1,-4,3)$, $(1,-5,2)$, $(1,-3,0)$ and whose centre lies on the plane $x + y + z = 0$

Solution

Mentioned points define some plane. Cross-section of plane and sphere is circle. As all points have the same x-coordinate, then circle lay on yz-plane. Hence, we can write

$$\begin{cases}(-4-y_0)^2+(3-z_0)^2=r^2,\\(-5-y_0)^2+(2-z_0)^2=r^2,\\(-3-y_0)^2+z_0^2=r^2,\end{cases}\rightarrow\begin{cases}(y_0+4)^2+(z_0-3)^2=r^2,\\(y_0+5)^2+(z_0-2)^2=r^2,\\(y_0+3)^2+z_0^2=r^2,\end{cases}$$

where y_0, z_0 are coordinates of the center of the circle and, at the same time, coordinates of the center of the sphere; r is a radius of the circle.

Solve system for y_0 and z_0:

$$\begin{cases}(y_0+4)^2+(z_0-3)^2=(y_0+3)^2+z_0^2\\(y_0+5)^2+(z_0-2)^2=(y_0+3)^2+z_0^2\end{cases}\rightarrow$$

$$\begin{cases}(2y_0+7)-3(2z_0-3)=0\\2(2y_0+8)-2(2z_0-2)=0\end{cases}\rightarrow\begin{cases}2y_0-6z_0+16=0\\4y_0-4z_0+20=0\end{cases}\rightarrow\begin{cases}y_0-3z_0+8=0\\y_0-z_0+5=0\end{cases}\rightarrow$$

$$2z_0-3=0\rightarrow z_0=\frac{3}{2}.\text{Then } y_0=z_0-5\rightarrow y_0=-\frac{7}{2}$$

x_0-coordinate can be found from the following condition:

$$x_0+y_0+z_0=0\rightarrow x_0=-y_0-z_0$$

$$x_0=\frac{7}{2}-\frac{3}{2}=\frac{4}{2}=2$$

Hence the center of the sphere is

$$\left(2,-\frac{7}{2},\frac{3}{2}\right).$$

The only thing left is to find squared radius of the sphere. Let's calculate the distance from the center to the 1st point:

$$R^2=(1-2)^2+\left(-4-\left(-\frac{7}{2}\right)\right)^2+\left(3-\frac{3}{2}\right)^2\rightarrow$$

$$R^2=(-1)^2+\left(-\frac{1}{2}\right)^2+\left(\frac{3}{2}\right)^2\rightarrow R^2=1+\frac{1}{4}+\frac{9}{4}=\frac{14}{4}$$

Thus, equation of the sphere is

$$(x-2)^2+\left(y+\frac{7}{2}\right)^2+\left(z-\frac{3}{2}\right)^2=\frac{14}{4}.$$

Answer: $(x-2)^2+\left(y+\frac{7}{2}\right)^2+\left(z-\frac{3}{2}\right)^2=\frac{14}{4}.$