

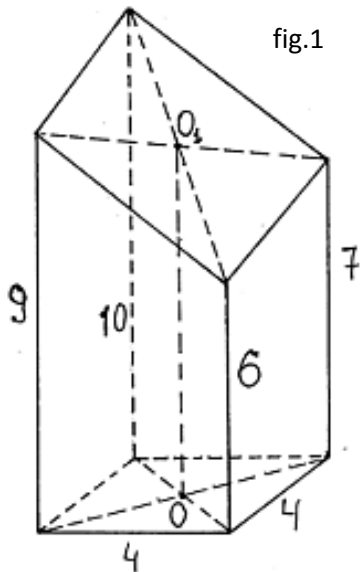
Answer on Question #65078 – Math – Geometry

Question

find the volume and lateral area of a truncated right square prism whose base edge is 4ft., and whose lateral edges measure 6ft., 7ft., 9ft., and 10ft.

Solution

Note that the opposite edges of the upper base must be parallel. Only in this case the upper base is the plane. This is possible if the lateral edges have dimensions as shown below (fig.1). In order to get the volume we add the prism by its mirror image to make a complete (non-



truncated) right prism with parallel square bases (fig. 2). The volume V_{nt} is twice of the given prism and equals [1]

$$V_{nt} = S_b \cdot H$$

where S_b is the area of the base, $S_b = 4^2 = 16 \text{ ft}^2$, H is the altitude of this prism. To find H we must add the lengths of the opposite edges:

$$H = 10 + 6 = 9 + 7 = 16$$

or

$$H = \frac{1}{2}(10 + 6 + 9 + 7) = 16$$

Then volume of a truncated prism is

$$V = \frac{1}{2}V_{nt} = \frac{1}{2}S_b \cdot H = S_b \cdot l$$

where

$$l = \frac{1}{2}H = \frac{1}{4}(6 + 9 + 10 + 7) = 8\text{ft}$$

Then we get

$$V = S_b \cdot l = 16 \cdot 8 = 128 \text{ ft}^3$$

Find the lateral area S_l . S_l is equal to the sum of the areas of the side faces. Each lateral face is a rectangular trapezoid, area S_t of which is defined by formula [1]

$$S_t = \frac{a + b}{2} \cdot h$$

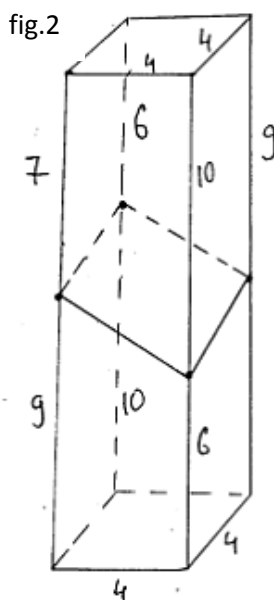
where a , b are the bases of trapezoid equal to the lateral edges, h is the altitude, which equals here 4 ft.

Then

$$S_l = \frac{1}{2}(6 + 9) \cdot 4 + \frac{1}{2}(9 + 10) \cdot 4 + \frac{1}{2}(10 + 7) \cdot 4 + \frac{1}{2}(7 + 6) \cdot 4$$

$$4S_l = (6 + 7 + 9 + 10) \cdot 4 = 32 \cdot 4 = 128 \text{ ft}^2$$

Answer: The volume of prism is $V = 128 \text{ ft}^3$, the lateral area is $S_l = 128 \text{ ft}^2$.



Reference:

[1] Daniel C. Alexander, Geralyn M. Koeberlein. *Elementary Geometry for College Students*, 6th ed.