Answer on Question #65075 – Math – Differential Equations

Solve the following differential equations

Question

i)

$$[D^3 - DD'^2 - D^2 + DD']z = 0$$

Solution

$$D(D^2 - D'^2 - D + D')z = 0$$

$$D[(D - D')(D + D') - (D - D')]z = 0$$

$$D(D - D')(D + D' - 1)z = 0$$

Corresponding for each non-repeated factor $(bD - aD' - c)$, the part of general solution is taken as

$$e^{cx/b}\varphi(by + ax) \quad ; \quad b \neq 0$$

Then

$$D' \implies \varphi_1(y)$$

$$(D - D') \implies \varphi_2(y + x)$$

$$(D + D' - 1) \implies e^x\varphi_3(y - x)$$

$$z = \varphi_1(y) + \varphi_2(y + x) + e^x\varphi_3(y - x)$$

Answer: $z = \varphi_1(y) + \varphi_2(y + x) + e^x\varphi_3(y - x)$.

Reference: *Ordinary And Partial Differential Equations, 18th Edition, Dr. M. D. Raisinghania, Section 5.6*

Question

ii)

$$[D^4 - D'^4 - 2D^2D'^2]z = 0$$

Solution

$(D^4 - D'^4 - 2D^2D'^2)$ cannot be resolved into linear factors.
Let a trial solution be:

$$z = Ae^{hx+ky} \quad ; \quad A, k, h - arbitrary\ constants$$

Then

$$Dz = Ahe^{hx+ky} \quad ; \quad D'z = Ake^{hx+ky}$$

$$D^2z = Ah^2e^{hx+ky} \quad ; \quad D'^2z = Ak^2e^{hx+ky}$$
$$D^4z = Ah^4e^{hx+ky} \quad ; \quad D'^4z = Ak^4e^{hx+ky}$$

$$[D^4 - D'^4 - 2D^2D'^2]z = Ae^{hx+ky}(h^4 - k^4 - 2h^2k^2) = 0$$

$$z = \sum Ae^{hx+ky} \quad ; \quad h^4 - k^4 - 2h^2k^2 = 0$$

Answer: $z = \sum Ae^{hx+ky}; \quad h^4 - k^4 - 2h^2k^2 = 0.$

Question

iii)

$$[D^2 - 2DD' + D'^2]z = 12xy$$

Solution

The auxiliary equation:

$$k^2 - 2k + 1 = 0$$

$$k_{1,2} = 1$$

Then the complementary function:

$$u = \varphi_1(y + 1, x) + x\varphi_2(y + 1, x) = \varphi_1(y + x) + x\varphi_2(y + x)$$

When $f(x, y) = V$, where V is function of x and y, then:

Partial integral:

$$p = \frac{1}{F(D, D')} V$$

We have $F(D, D')$ and $V = 12xy$.

Then partial integral:

$$p = \frac{1}{D^2 - 2DD' + D'^2}(12xy) = \frac{1}{(D - D')^2}(12xy) = \frac{1}{D^2\left(1 - \frac{D'}{D}\right)^2}(12xy) =$$

$$= \frac{12}{D^2}\left(1 - \frac{D'}{D}\right)^{-2}(xy) = \frac{12}{D^2}\left(1 + \frac{2D'}{D} + \frac{3D'}{D^2} + \cdots\right)(xy) = \frac{12}{D^2}\left(xy + \frac{2}{D}(x) + 0 \ldots\right) =$$

$$= \frac{12}{D^2}(xy) + \frac{24}{D^3}(x) = 12y\left(\frac{x^3}{6}\right) + 24\left(\frac{x^4}{24}\right) = x^4 + 2x^3y;$$

$$z = u + p = \varphi_1(y + x) + x\varphi_2(y + x) + x^4 + 2x^3y,$$

where φ_1, φ_2 are arbitrary functions.

Answer: $z = u + p = \varphi_1(y + x) + x\varphi_2(y + x) + x^4 + 2x^3y,$

where φ_1, φ_2 are arbitrary functions.

Reference: *Ordinary And Partial Differential Equations, 18th Edition, Dr. M. D. Raisinghania, Section 5.7*