Answer on Question #64971 – Math – Trigonometry

Question

In any triangle *ABC*, if sin² *A*= sin²*B* + sin²*C*, prove that the triangle is right angled at A.

Solution

By the sine law [1, p. 72],

$$\frac{\sin A}{BC} = \frac{\sin B}{AC} = \frac{\sin C}{AB} = d.$$

From this formula we get

$$\sin A = d \cdot BC, \sin B = d \cdot AC, \sin C = d \cdot AB.$$

We have

$$d^2 \cdot BC^2 = \sin^2 A = \sin^2 B + \sin^2 C = d^2 \cdot AC^2 + d^2 \cdot AB^2,$$

and

$$BC^2 = AC^2 + AB^2.$$

By the converse of the Pythagorean theorem [1, p.7], it follows from the last equality that the triangle *ABC* is right angled at *A*.

Reference:

Gelfand, M., & Saul, M. (2001) *Trigonometry*. Birkhäuser Boston. 10.1007/978-1-4612-0149-6