Answer on Question #64892 – Math – Linear Algebra

Question

Which of the following are binary operations on S = {x ∈ R | x > 0}? Justify your answer.

i) The operation Δ defined by xΔy = x(y− 2).

ii) The operation ∇ defined by x∇y = e^x+y.

Also, for those operations which are binary operations, check whether they are associative and commutative.

Solution

S = {x ∈ R | x > 0}.

i. $x\Delta y = x(y-2)$

Binary operation: $S\times S \to S$, so for any $x \in S, y \in S, x(y-2)$ must be an element of S.

So $x(y-2)$ must be positive for any positive x, y. If $0 < y < 2$, then $x(y-2) < 0 \Rightarrow x\Delta y \notin S$

So $x\Delta y = x(y-2)$ is not a binary operation on **S = {x ∈ R | x > 0};**

ii. $x\nabla y = e^x + y$

Binary operation: $S\times S \to S$, so for any $x \in S, y \in S, e^x + y$ must be an element of S.

That is true, because for any positive x, y - $e^x + y$ is also positive ⇒ $x\Delta y \in S$.

So $x\nabla y = e^x + y$ is a binary operation on **S = {x ∈ R | x > 0};**

Associativity:

($(x\nabla y)\nabla z = x\nabla(y\nabla z)$)

$(x\nabla y)\nabla z = (e^x + y)\nabla z = e^{e^x+y} + z = e^{e^x} * e^y + z$**;**

$x\nabla(y\nabla z) = x\nabla(e^y + z) = e^x + e^y + z;$

$e^{e^x} * e^y + z \neq e^x + e^y + z \Rightarrow (x\nabla y)\nabla z \neq x\nabla(y\nabla z)$

Operation is not associative.

Commutativity:

($x\nabla y = y\nabla x$)

$(x\nabla y) = (e^x + y)$**;**

$(y\nabla x) = (e^y + x);$

$e^x + y \neq e^y + x \Rightarrow (x\nabla y) \neq (y\nabla x)$

Operation is not commutative.

Answer: ii) is a binary relation, not associative, not com mutative.