Answer on Question #64865 – Math – Real Analysis

Question

Compute the Riemann integral of the function $f(x) = x$ on the interval [−1, 1].

Solution

Theorem 1. If a function f is continuous on an interval $[a,b]$, it is also Riemann-integrable on this interval.

Note that $f(x) = x$ is elementary function, then it is continuous on its domain, so $f(x) = x$ is integrable on the interval [−1, 1] and integral $\int_{-1}^{1} f(x)dx$ exists.

Compute this integral.

Method 1

Function is odd if $f(-x) = -f(x)$. Note that $f(-x) = -x = -f(x)$, hence $f(x) = x$ is odd.

Theorem 2. Let the real function f be Riemann-integrable on $[-a, a]$ and if f is an odd function, then $\int_{-a}^{a} f(x)dx = 0$.

By Theorem 2 we get $\int_{-1}^{1} f(x)dx$ =0.

Method 2

By the Table of Integrals

$$\int xdx = \frac{x^2}{2} + C$$

and

Newton-Leibniz rule

$$\int_{a}^{b} f(x)dx = F(b) - F(a) = F(x)\Big|_{a}^{b}, \ where\ F' = f\ ,$$

we get

$$\int_{-1}^{1} f(x)dx = \int_{-1}^{1} xdx = \frac{x^2}{2}\Big|_{-1}^{1} = \frac{1^2}{2} - \frac{(-1)^2}{2} = \frac{1}{2} - \frac{1}{2} = 0\,.$$

Answer: 0.