

## Answer on Question #64863 – Math – Real Analysis

### Question

Verify Inverse function theorem for finding the derivative at a point  $y$  of the 0 domain of the inverse function of the function  $f(x) = \cos x$ ,  $x \in [0, \pi]$ . Hence, find the derivative of the inverse function at  $y$ .

### Solution

Derivative of the Inverse Function:

$$\frac{d}{dx} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}.$$

$$f(x) = \cos x, \quad f'(x) = -\sin x, \quad f^{-1}(x) = \arccos x.$$

$$y = f(x) = 0 \text{ when } x = \frac{\pi}{2}.$$

Thus

$$\frac{d}{dx} f^{-1}(x)|_{x=0} = \frac{1}{-\sin(\arccos(0))} = -\frac{1}{\sin\left(\frac{\pi}{2}\right)} = -1.$$

**Answer:**  $-1$ .