Answer on Question #64817 – Math – Algebra

Question

In (triangle) ABC, <C is a right angle, a=3, c=19. Find the remaining sides and angles. Round your answers to the nearest tenth. Show your work.

Solution

If $a = 3, c = 19$

$$b = \sqrt{c^2 - a^2} = \sqrt{19^2 - 3^2} = \sqrt{352} \approx 18.8.$$

$$sinA = \frac{a}{c} = \frac{3}{19} \quad \rightarrow \quad A = sin^{-1}\left(\frac{3}{19}\right) \approx 9.1^0.$$

$$B = 90^0 - A = 90^0 - 9.1^0 = 80.9^0.$$

Answer: 18.8; 9.1^0; 80.9^0.

Question

Find the values of the 30th and 90th percentiles of the data. Show your work. 129, 113, 200, 100, 105, 132, 100, 176, 146, 152

Solution

Ordered data:

$100,\ 100,\ 105,\ 113,\ 129,\ 132,\ 146,\ 152,\ 176,\ 200.$

Find out what rank is at the 30th percentile by means of the following formula

$$Rank = \frac{Percentile}{100} \cdot (number\ of\ items + 1) = \frac{30}{100} \cdot 11 = 3.3$$

As 3.3 is closer to 3 than 4, one will round down to a rank of 3

There exists three ways of definition of the 30th percentile.

The most popular is Definition 1.

Definition 1. The 30th percentile is the lowest score that is greater than 30% of the score. That has a rank of 4 and equals a score of 113.

Definition 2. The 30th percentile is the lowest score that is greater than or equal to 30% of the scores. That has a rank of 3 and equals a score of 105.

Definition 3. The 30th percentile is a weighted mean of the percentiles from the first two definitions.

As 3.3 is between 3 and 4, the scores were 105 and 113, the fraction of the rank calculated above is 0.3, one should multiply the difference between the scores by 0.3:

$$0.3 \cdot (113 - 105) = 0.3 \cdot 8 = 2.4$$

Finally add the result to the lower score

$$105 + 2.4 = 107.4$$

Find out what rank is at the 90th percentile by means of the following formula

$$Rank = \frac{Percentile}{100} \cdot (number\ of\ items + 1) = \frac{90}{100} \cdot 11 = 9.9$$

As 9.9 is closer to 10 than 9, one will round to a rank of 10.

The 90th percentile is 200.

Answer: 113; 200

Question

Verify the identity. Justify each step.

sec 0 sec 0

___________ - ___________=2 csc 0

csc 0-cot 0 csc 0+cot 0

Solution

$$\frac{sec\theta}{csc\theta - cot\theta} - \frac{sec\theta}{csc\theta + cot\theta} = sec\theta\left(\frac{1}{csc\theta - cot\theta} - \frac{1}{csc\theta + cot\theta}\right)$$

$$= \frac{sec\theta(csc\theta + cot\theta - csc\theta + cot\theta)}{csc^2\theta - cot^2\theta} == \frac{sec\ \theta \cdot 2cot\theta}{\frac{1 - cos^2\theta}{sin^2\theta}} = \frac{\frac{2cos\theta}{sin^2\theta}}{\frac{sin^2\theta}{sin^2\theta}}$$

$$= \frac{\frac{2cos\theta}{sin^2\theta}}{1} = \frac{2cos\theta}{sin^2\theta} = \frac{2}{sin\theta} \cdot \frac{cos\theta}{sin\theta};$$

Using the definition of the cosecant

$$2csc\theta = \frac{2}{sin\theta}.$$

It follows from the previous formulas that

$$\frac{sec\theta}{csc\theta - cot\theta} - \frac{sec\theta}{csc\theta + cot\theta} = 2csc\theta \quad \text{if and only if} \quad \frac{cos\theta}{sin\theta} = 1,$$

that is,

$$\theta = \frac{\pi}{4} + n\pi, n \in \mathbb{Z} .$$

It is not identity. It is an equation. Its roots are $\theta = \frac{\pi}{4} + n\pi, n \in \mathbb{Z}$.

Answer: $\theta = \frac{\pi}{4} + n\pi, n \in \mathbb{Z}$.