

Answer on Question #64793 - Math – Calculus

Question

How do I find the recursive definition of an arithmetic sequence

Solution

An arithmetic sequence is a sequence of numbers such that the difference between the consecutive terms is constant. Hence each term of the arithmetic sequence is computed from the previous one by adding a constant d .

The recursive formula is

$$a_n = a_{n-1} + d \quad (1)$$

The previous formula can be rewritten as

$$\begin{aligned} a_n &= a_{n-1} + d = a_{n-2} + 2d = a_{n-3} + 3d = \dots = a_{n-(n-1)} + (n-1)d = \\ &= a_1 + (n-1)d. \end{aligned}$$

If the initial term of an arithmetic progression is a_1 and the common difference of successive terms is d , then the n th term of the sequence (a_n) is given by the explicit formula:

$$a_n = a_1 + (n-1)d. \quad (2)$$

It follows from the formula (2) that

$$a_m = a_1 + (m-1)d. \quad (3)$$

Subtracting (3) from (2) one gets

$$\begin{aligned} a_n - a_m &= a_1 + (n-1)d - (a_1 + (m-1)d) = (n-1)d - (m-1)d = \\ &= (n-m)d, \end{aligned}$$

hence

$$a_n = a_m + (n-m)d \quad (4)$$

If $m = n - 1$, then the formula (4) gives the formula (1).

If $m = 1$, then the formula (4) gives the formula (2).

Answer: $a_n = a_{n-1} + d$.