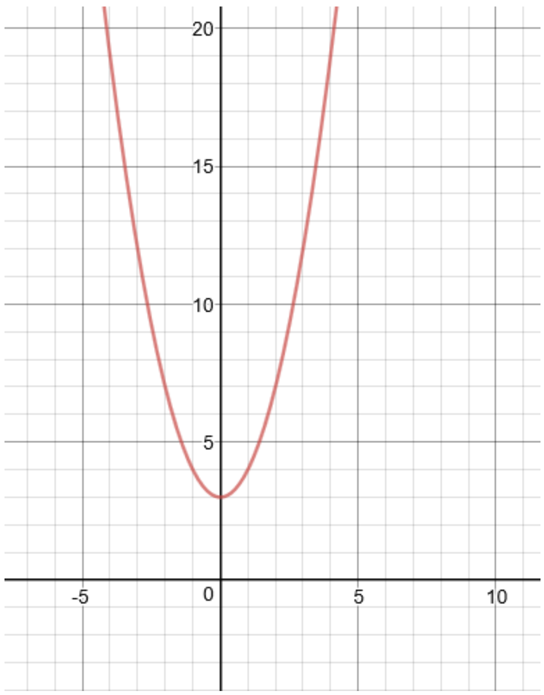
Answer on Question #64791 – Math – Calculus

Question

For each of the following curves, make a sketch of the curve and find:
I. the gradient function
II. the gradient of the tangent at the given point.



a. $y = x^2 + 3$ at point $(2,7)$

Solution

I. The gradient function gives the slope of a function at any single point on its curve. The slope of a curve at a point is defined to be the slope of the tangent line. Hence the slope of a curve at a point is found using the derivative.
The slope of a function $y = f(x)$ at a point $(x_0, f(x_0))$ is given by

$$m = f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h)-f(x_0)}{h}.$$

Thus, the gradient function is

$$\nabla y = \frac{dy}{dx} = (x^2+3)' = (x^2)' + 3' = 2x + 0 = 2x$$

II.

Method 1

The gradient of the tangent at the given point $\left(x_0, f(x_0)\right) = (2,7)$ is

$$f'(x_0) = f'(2) = 2 \cdot 2 = 4.$$

Method 2

Equation of the tangent at the given point $\left(x_0, f(x_0)\right)$ is given by

$$y - y_0 = f'(x_0)(x - x_0)$$

Substituting

$$x_0 = 2, f(x_0) = 7,$$
$$f'(x_0) = f'(2) = 2 \cdot 2 = 4,$$

one gets

$$y - 7 = 4(x - 2)$$

$$y = 4x - 1$$

The gradient of the tangent at the given point $\left(x_0, f(x_0)\right) = (2,7)$ is the slope of $y = 4x - 1$:

$$m = 4$$

Answer: **I.** $2x$. **II.** 4.

Question

For each of the following curves, make a sketch of the curve and find:
I. the gradient function
II. the gradient of the tangent at the given point.

b. $y = x^2 + k$ at point $(1, 1 + k)$

Solution

I.

The gradient function gives the slope of a function at any single point on its curve. The slope of a curve at a point is defined to be the slope of the tangent line.
Hence the slope of a curve at a point is found using the derivative.
The slope of a function $y = f(x)$ at a point $\left(x_0, f(x_0)\right)$ is given by

$$m = f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}.$$

Thus, the gradient function is

$$\nabla y = \frac{dy}{dx} = (x^2 + k)' = (x^2)' + k' = 2x + 0 = 2x$$

II.

Method 1

The gradient of the tangent at the given point $\left(x_0, f(x_0)\right) = (1, 1 + k)$ is

$$f'(x_0) = f'(1) = 2 \cdot 1 = 2.$$

Method 2

Equation of tangent at the given point $\left(x_0, f(x_0)\right)$ is given by

$$y - y_0 = f'(x_0)(x - x_0)$$

Substituting

$$x_0 = 1, f(x_0) = 1 + k$$

$$f'(x_0) = 2 \cdot 1 = 2$$

one gets

$$y - (1 + k) = 2(x - 1)$$

$$y = 2x - 1 + k$$

The gradient of the tangent at the given point $\left(x_0, f(x_0)\right) = (1, 1 + k)$ is the slope of $y = 2x - 1 + k$:

$$m = 2$$

Answer: **I.** $2x$. **II.** 2.