Answer on Question #64620 – Math – Differential Equations

Question

Solve the following differential equation:

$$(D3 + D2 - 4D + 4)y = e^{2x}$$

Solution

Consider the following problem:

$$(D3 - D2 - 4D + 4)y = e^{2x}$$

The reduced equation is

$$(D^3 - D^2 - 4D + 4)y = 0$$

Let $y = Ae^{mx}$ be a trial solution of reduced equation and then the auxiliary equation is

$$m^3 - m^2 - 4m + 4 = 0;$$
$$(m-1)(m-2)(m+2) = 0;$$
$$m = -2, 1, 2.$$

The complementary function is

$$y = c_1 e^{2x} + c_2 e^{x} + c_3 e^{-2x}.$$

$Q(x) = e^{2x}$.

A particular solution to the inhomogeneous equation is

$$x \cdot \frac{1}{f'(2)} e^{2x}.$$

Compute

$$(D^3 - D^2 - 4D + 4)' = 3D^2 - 2D - 4$$
$$f'(2) = 3 \cdot 2^2 - 2 \cdot 2 - 4 = 4.$$

Thus

$$x \cdot \frac{1}{f'(2)} e^{2x} = \frac{1}{4} x e^{2x}.$$

Hence the general solution is

$$y = c_1 e^{2x} + c_2 e^{x} + c_3 e^{-2x} + \frac{1}{4} x e^{2x}.$$

Answer: $y = c_1 e^{2x} + c_2 e^{x} + c_3 e^{-2x} + \frac{1}{4} x e^{2x}$.