Answer on Question #64605 – Math – Calculus

Question

23. An open-topped cylindrical pot is to have volume 250 cm^3.The material for the bottom of the pot costs 4 cents per cm^2; that for its curved side costs 2cents per cm^2. What dimensions will minimize the total cost of this pot?

Solution

Let r be the radius of the pot and h be its height. Then the volume is given by

$$V = \pi r^2 h => h = \frac{V}{\pi r^2}.$$

The area of the bottom of the pot is

$$A_1 = \pi r^2 .$$

So it will cost $4\pi r^2$ to make it.
The area of the side of the pot is

$$A_2 = 2\pi r h = 2\pi r \frac{V}{\pi r^2} = \frac{2V}{r}.$$

So it will cost $4V/r$ to make it. The total cost is

$$C(r) = 4\pi r^2 + \frac{4V}{r} = 4\pi r^2 + \frac{1000}{r}.$$

We want to minimize this cost. Find the derivative

$$C'(r) = 8\pi r - \frac{1000}{r^2}$$

The critical points are the roots of the following equation:

$$C'(r) = 0 => 8\pi r - \frac{1000}{r^2} = 0 .$$

$$r^3 = \frac{125}{\pi};$$

$$r = \frac{5}{\sqrt[3]{\pi}}\ (cm) .$$

Since the derivative changes sign at $r = 5/\sqrt[3]{\pi}$ from − to +, this is a point of local minimum. Since

$$\lim_{r\to 0^+} C(r) = \infty, \lim_{r\to\infty} C(r) = \infty ,$$

the point $r = 5/\sqrt[3]{\pi}$ is also a point of global minimum, and so the optimal radius should be $r = 5/\sqrt[3]{\pi}$ (cm). The corresponding height is

$$h = \frac{250 \cdot \sqrt[3]{\pi^2}}{\pi \cdot 25} = \frac{10}{\sqrt[3]{\pi}}\ (cm).$$

Answer: $r = \frac{5}{\sqrt[3]{\pi}}\ cm,\ h = \frac{10}{\sqrt[3]{\pi}}\ cm.$

Question

24. A storage container is to be made in the form of a right circular cylinder and have a volume of 28π m^3. Material for the top of the container costs \$5 per square metre and material for the side and base costs \$2 per square metre. What dimensions will minimize the total cost of the container?

Solution

Let r be the radius of the container and h be its height. Then the volume is given by

$$V = \pi r^2 h \Rightarrow h = \frac{V}{\pi r^2}.$$

The area of the top of the container is

$$A_1 = \pi r^2 .$$

So it will cost $5\pi r^2$ to make it.

The area of the bottom of the container is

$$A_2 = \pi r^2 .$$

So it will cost $2\pi r^2$ to make it.

The area of the side of the container is

$$A_3 = 2\pi r h = 2\pi r \frac{V}{\pi r^2} = \frac{2V}{r}.$$

So it will cost $4V/r$ to make it. The total cost is

$$C(r) = 5\pi r^2 + 2\pi r^2 + \frac{4V}{r} = 7\pi r^2 + \frac{112\pi}{r}.$$

We want to minimize this cost. Find the derivative

$$C'(r) = 14\pi r - \frac{112\pi}{r^2}$$

The critical points are the roots of the following equation:

$$C'(r) = 0 \Rightarrow 14\pi r - \frac{112\pi}{r^2} = 0 .$$

$$r^3 = 8;$$

$$r = 2\ (m).$$

Since the derivative changes sign at $r = 2$ from $-$ to $+$, this is a point of local minimum. Since

$$\lim_{r\to 0^+} C(r) = \infty, \lim_{r\to\infty} C(r) = \infty ,$$

the point $r = 2$ is also a point of global minimum, and so the optimal radius should be $r = 2\ m$. The corresponding height is

$$h = \frac{28\pi}{\pi\cdot 2^2} = 7\ (m).$$

Answer: $r = 2\ m,\ h = 7\ m.$