Answer on Question #64604 – Math – Calculus

Question

19. A rectangular box has a square base with edge length x of at least 1 unit. The total surface area of its six sides is 150 square units.

(a) Express the volume V of this box as a function of x.

(b) Find the domain of V (x).

(c) Find the dimensions of the box in part (a) with the greatest possible volume. What is this greatest possible volume?

Solution

(a) Let y be the height. The total surface area is

$$A = 2x^2 + 4xy = 150,$$

hence

$$y = \frac{150 - 2x^2}{4x} = \frac{75 - x^2}{2x}.$$

The volume is

$$V = x^2 y = x^2 \frac{75 - x^2}{2x} = \frac{x}{2}(75 - x^2).$$

(b) Given $x \geq 1$. Besides

$$y > 0 \rightarrow \frac{75 - x^2}{2x} > 0$$

$$75 > x^2$$

$$x < \sqrt{75} = 5\sqrt{3}$$

Thus, the domain of V (x) is $\left[1; 5\sqrt{3}\right)$.

(c)

$$\frac{dV}{dx} = \frac{d}{dx}\left(\frac{75x}{2} - \frac{x^3}{2}\right) = \frac{75}{2} - \frac{3x^2}{2} = 0.$$

The dimensions of the box are

$$25 = x^2 \rightarrow x = 5$$

$$y = \frac{75 - 25}{2 \cdot 5} = 5$$

The greatest possible volume is

$$V(5) = \frac{5}{2}(75 - 25) = 125.$$

Answer: (a) $\frac{x}{2}(75 - x^2)$; **(b)** $[1; 5\sqrt{3})$; **(c)** 5 units, 5 units; 125 units^3.

Question

20. An open-top box is to have a square base and a volume of 13500 cm3. Find the dimensions of the box that minimize the amount of material used.

Solution

The area of a square base is

$$x^2.$$

The volume is

$$V = x^2 h = 13500$$

$$h = \frac{13500}{x^2}.$$

The amount of material used is proportional to the surface area:

$$A = x^2 + 4xh = x^2 + 4x\frac{13500}{x^2} = x^2 + \frac{54000}{x}.$$

$$\frac{dA}{dx} = 2x - \frac{54000}{x^2} = 0$$

$$x^3 = \frac{54000}{2} = 27000.$$

The length and width are

$$x = \sqrt[3]{27000} = 30\ cm.$$

The height is

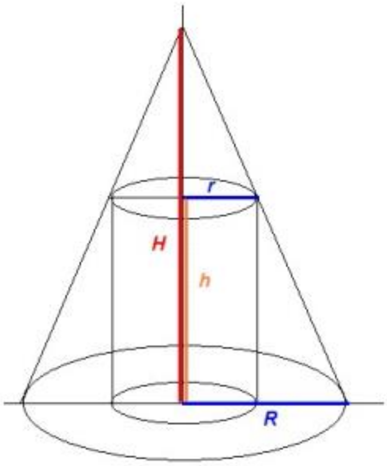
$$h = \frac{13500}{30^2} = 15\ cm.$$

Answer: 30 cm, 30 cm, 15 cm.

Question

21. Find the dimension of the right circular cylinder of maximum volume that can be inscribed in a right circular cone of radius R and height H.

Solution



$$\frac{H}{R} = \frac{h}{R - r}$$

$$h = \frac{H(R - r)}{R}.$$

The volume of cylinder is

$$V = \pi r^2 h = \pi r^2 \frac{H(R - r)}{R} = \frac{\pi H}{R}(r^2 R - r^3).$$

$$\frac{dV}{dr} = \frac{\pi H}{R}(2rR - 3r^2) = 0$$

$$2R = 3r$$

The radius of cylinder is

$$r = \frac{2}{3}R.$$

The height of cylinder is

$$h = \frac{H\left(R - \frac{2}{3}R\right)}{R} = \frac{1}{3}H.$$

Answer: $r = \frac{2}{3}R,\ h = \frac{1}{3}H.$

Question

22. A hollow plastic cylinder with a circular base and open top is to be made and 10 m2 plastic is available. Find the dimensions of the cylinder that give the maximum volume and find the value of the maximum volume.

Solution

The area of a base is

$$\pi r^2.$$

The volume is

$$V = \pi r^2 h$$

The amount of material used is proportional to the surface area:

$$A = \pi r^2 + 2\pi r h = 10.$$

$$h = \frac{10 - \pi r^2}{2\pi r}$$

So,

$$V = \pi r^2 \left(\frac{10 - \pi r^2}{2\pi r}\right) = \frac{r}{2}(10 - \pi r^2).$$

$$\frac{dV}{dr} = \frac{10}{2} - \frac{3\pi r^2}{2} = 0$$

$$3\pi r^2 = 10$$

The radius is

$$r = \sqrt{\frac{10}{3\pi}}\ m.$$

The height is

$$h = \frac{10 - \pi \frac{10}{3\pi}}{2\pi\sqrt{\frac{10}{3\pi}}} = \sqrt{\frac{10}{3\pi}}\ m.$$

The value of the maximum volume is

$$V_{max} = \frac{r}{2}(10 - \pi r^2) = \frac{\sqrt{\frac{10}{3\pi}}}{2}\left(10 - \pi \cdot \frac{10}{3\pi}\right) = \frac{10}{3}\sqrt{\frac{10}{3\pi}}\, m^3.$$

Answer: $r = \sqrt{\frac{10}{3\pi}}\ m.$, $h = \sqrt{\frac{10}{3\pi}}\ m,\ V_{max} = \frac{10}{3}\sqrt{\frac{10}{3\pi}}\, m^3.$