Answer on Question #64602 – Math – Calculus

Question

13. A boy starts walking north at a speed of 1.5 m/s, and a girl starts walking west at the same point P at the same time at a speed of 2 m/s. At what rate is the distance between the boy and the girl increasing 6 seconds later?

14. A police car, approaching right-angled intersection from the north, is chasing a speeding SUV that has turned the corner and is now moving straight east. When the police car is 0.6 km north of intersection and the SUV is 0.8 km east of intersection, the police determine with radar that the distance between them and the SUV is increasing at 20 km/hr. If the police car is moving at 60 km/hr at the instant of measurement, what is the speed of the SUV?

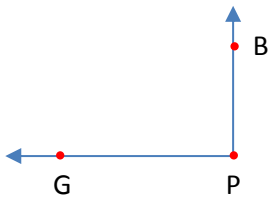
15. A farmer has 400 feet of fencing with which to build a rectangular pen. He will use part of an existing straight wall 100 feet long as part of one side of the perimeter of the pen. What is the maximum area that can be enclosed?.

Solution

13.

$D(t) = \sqrt{[B(t)]^2 + [G(t)]^2},\ D'(6) = ???.$

$B'(t) = 1.5,\ G'(t) = 2,\ B(6) = 1.5 * 6 = 9,\ G(6) = 2 * 6 = 12.$



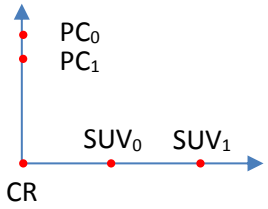
$[D(t)]^2 = [B(t)]^2 + [G(t)]^2 \Longrightarrow 2 * D(t) * D'(t) = 2 * B(t) * B'(t) + 2 * G(t) * G'(t) \Longrightarrow$

$D'(t) = \big(B(t) * B'(t) + G(t) * G'(t)\big)/D(t).$

So, $D'(6) = \big(B(6) * B'(6) + G(6) * G'(6)\big)/D(6) = (9 * 1.5 + 12 * 2)/\sqrt{[12]^2 + [9]^2} = 2.5.$

14. $D(t) = \sqrt{[PC(t)]^2 + [SUV(t)]^2},\ SUV'(1) = ???.$

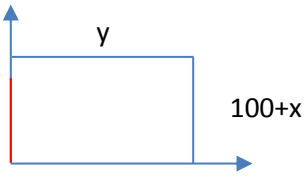
$PC(1) = 0.6,\ SUV(1) = 0.8,\ PC'(1) = -60$ (i.e. descreasing), $D'(1) = 20$ (i.e. increasing).



$D'(t) = 1/2\,([PC(t)]^2 + [SUV(t)]^2)^{-1/2}(2 * PC(t) * PC'(t) + 2 * SUV(t) * SUV'(t)).$
$SUV'(t) = (D(t) * D'(t) - PC(t) * PC'(t))/SUV(t)$
$D(1) = \sqrt{[PC(1)]^2 + [SUV(1)]^2} = \sqrt{(0.6)^2 + (0.8)^2} = 1$

So, $SUV'(1) = (1 * 20 - 0.6 * (-60))/0.8 = 70.$

15. $A(x) = y(x) * x,\ y = (400 - x - 100 - x)/2,\ 0 \le x \le 400,\ 0 \le y \le 400,\ A_{max} = ???.$



$A'(x) = [(150 - x) * x]' = 150 - 2x.$
$x_1 = 75,\ A'(x = x_1) = 0,\ A'(x < x_1) = +,\ A'(x > x_1) = -.$
$0 \le x_1 \le 400,\ 0 \le y(x_1) \le 400.$

So, $A_{max} = A(x_1) = (150 - 75) * 75 = 5625.$

Answer:

13. The distance between the boy and the girl increasing at 2.5 m.
14. The speed of the SUV is 70 km/hr.
15. The maximum area that can be enclosed is 5625 square feets.