Answer on Question #64600 – Math – Calculus

Question

7. A helicopter takes off from a point 80 m away from an observer located on the ground, and rises vertically at 2 m/s. At what rate is elevation angle of the observer's line of sight to the helicopter changing when the helicopter is 60 m above the ground.

Solution

Let α = α(t) be the elevation angle, X be the distance, Y be the height.

$$\frac{dY}{dt} = 2\text{m/s},$$

$$\tan(\alpha) = \frac{Y}{X},$$

$$X*\tan(\alpha) = Y.$$

Differentiating Implicitly Over Time:

$$X*(1/cos^2\ (\alpha))*\frac{d\alpha}{dt} = \frac{dY}{dt}$$

$$\frac{d\alpha}{dt}=\frac{dY}{dt}*(cos^2(\alpha)/X) = 2*(cos^2(\alpha)/80) = cos^2(\alpha)/40$$

When t = 30 we have cos(α) = 4/5 .

$$\frac{d\alpha}{dt} = (16/25)*(1/40) = 0.016$$

Thus when the helicopter is 60 m above the ground the elevation angle of the observer's line of sight to the helicopter is increasing at a rate of 0.016 deg/sec.

Answer: 0.016 deg/sec.

Question

8. An oil slick on a lake is surrounded by a floating circular containment boom. As the boom is pulled in, the circular containment boom. As the boom is pulled in, the circular containment area shrinks (all the while maintaining the shape of a circle.) If the boom is pulled in at the rate of 5 m/min, at what rate is the containment area shrinking when it has a diameter of 100m?

Solution

Let **R** denote the radius of the circular containment area.

R = R(t)

It is given that $\frac{dR}{dt}$ = 5 m/min.

From the fact that the area at time **t** is given by

$$\mathbf{S} = \pi R^2(t).$$

Differentiating Implicitly Over Time:

$$\frac{dS}{dt} = 2*R*\pi*\frac{dR}{dt} = 10r*\pi*m^2/\text{min}.$$

When r = 50m then the area shrinks at the rate of 10 · 50 · π = 500π m^2/min.

Answer: 500π m^2/min.

Question

9. Consider a cube of variable size. (The edge length is increasing.) Assume that the volume of the cube is increasing at the rate of 10 cm3/minute. How fast is the surface area increasing when the edge length is 8 cm?

Solution

Let X = X(t) be the edge length.

Then the volume is given by V = X^3 and the surface area is given by S = 6X^2 .

It is given that

$$\frac{dV}{dt} = 10.$$

Differentiating Implicitly Over Time:

$$\frac{dV}{dt} = 3X^2\frac{dX}{dt} \qquad (1)$$

and

$$\frac{dS}{dt} = 12X\frac{dX}{dt} \qquad (2)$$

It follows from (1) that

$$\frac{dX}{dt} = \frac{\frac{dV}{dt}}{3X^2} = \frac{10}{3X^2}.$$

When X = 8:

$$\frac{dX}{dt} = 10/(3*25) = 5/96 \text{ cm/min} \qquad (3)$$

It follows from (2) and (3) that

$$\frac{dS}{dt} = 12*X\frac{dX}{dt} = 12*8*5/96 = 5\ cm^2/\text{min}$$

The surface area is increasing at the rate 5 cm^2/min.

Answer: 5 cm^2/min.