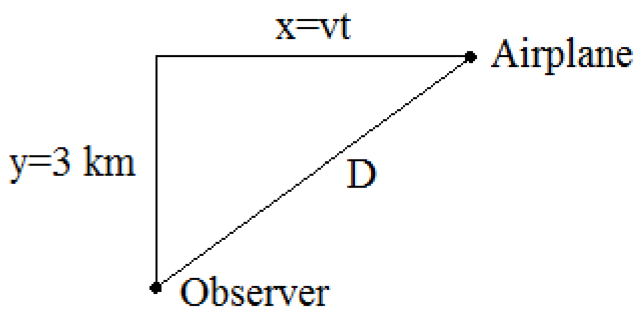
Answer on Question #64599 – Math – Calculus

Question

4. An airplane flying horizontally at an altitude of y = 3 km and at a speed of 480 km/h passes directly above an observer on the ground. How fast is the distance D from the observer to the airplane increasing 30 seconds later?

Solution



D is the distance from the airplane to the observer and x is the (horizontal) distance traveled by the airplane from the moment it passed over the observer.

We know that v $= dx/dt = 480\ km/h$.

We want to know dD/dt 30 seconds after the plane flew over the observer.

$$t = 30\ sec = 30 \cdot \frac{1}{3600}\ hours = \frac{1}{120}\ hours$$

Applying the Pythagorean Theorem to a right triangle

$$D^2 = x^2 + y^2, D^2 = x^2 + 3^2$$

Taking derivatives with respect to time t on both sides we get

$$2D\frac{dD}{dt} = 2x\frac{dx}{dt},$$

so that

$$\frac{dD}{dt} = \frac{x}{D} \cdot \frac{dx}{dt}$$

We have that

$$t = 30\ sec = \frac{1}{120}\ hours, \frac{dx}{dt} = \text{v} = 480\frac{km}{h}, x = \text{v}t = 480 \cdot \frac{1}{120} = 4\ (km),$$
$$D^2 = 4^2 + 3^2 = 25\ (km^2), D = 5\ km.$$

Then

$$\frac{dD}{dt} = \frac{4}{5} \cdot 480 = 384\ \left(\frac{km}{h}\right).$$

Answer: $384\frac{km}{h}$.

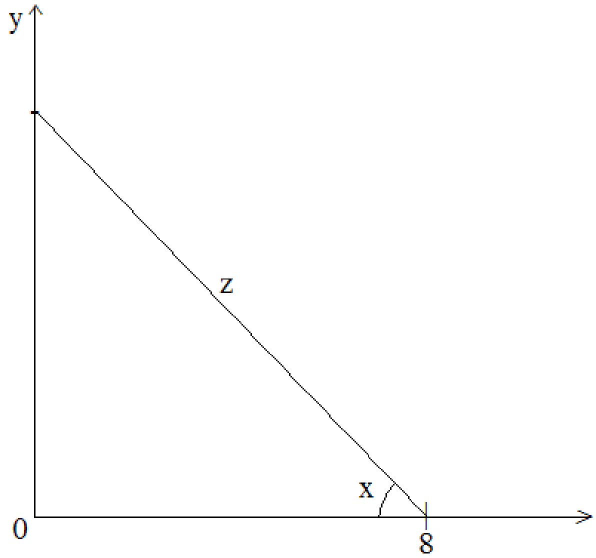
Question

5. A kite is rising vertically at a constant speed of 2 m/s from a location at ground level which is 8 m away from the person handling the string of the kite.

(a) Let z be the distance from the kite to the person. Find the rate of change

of z with respect to time t when z = 10.
(b) Let x be the angle the string makes with the horizontal. Find the rate of change of x with respect to time t when the kite is y = 6 m above ground.

Solution



(a) Applying the Pythagorean theorem to a right triangle
$$z^2 = 8^2 + y^2$$
We know that
$$v_y = \frac{dy}{dt} = 2\frac{m}{s}, y = v_y t\ .$$
Then
$$z^2 = 64 + (v_y t)^2$$
Taking derivatives with respect to time t on both sides we get
$$2z\frac{dz}{dt} = 2v_y^2 t.$$
So that
$$\frac{dz}{dt} = \frac{1}{z}v_y^2 t = \frac{y}{z}v_y\ .$$
We have that
$$z = 10m, 10^2 = 64 + y^2, y = 6\ m.$$
Therefore
$$\frac{dz}{dt} = \frac{6}{10}\cdot 2 = 1.2\ \left(\frac{m}{s}\right).$$
(b) Using the definition
$$\tan x = \frac{y}{8}$$
Taking derivatives with respect to time t on both sides we get
$$\frac{1}{\cos^2 x}\cdot\frac{dx}{dt} = \frac{1}{8}\cdot\frac{dy}{dt}.$$
So that
$$\frac{dx}{dt} = \frac{1}{8}\cdot\cos^2 x\cdot v_y.$$

We have that

$$y = 6\,m, \tan x = \frac{6}{8} = \frac{3}{4}, 1 + \tan^2 x = 1 + \left(\frac{3}{4}\right)^2 = \frac{25}{16} = \frac{1}{\cos^2 x}, \cos^2 x = \frac{16}{25}.$$

Therefore

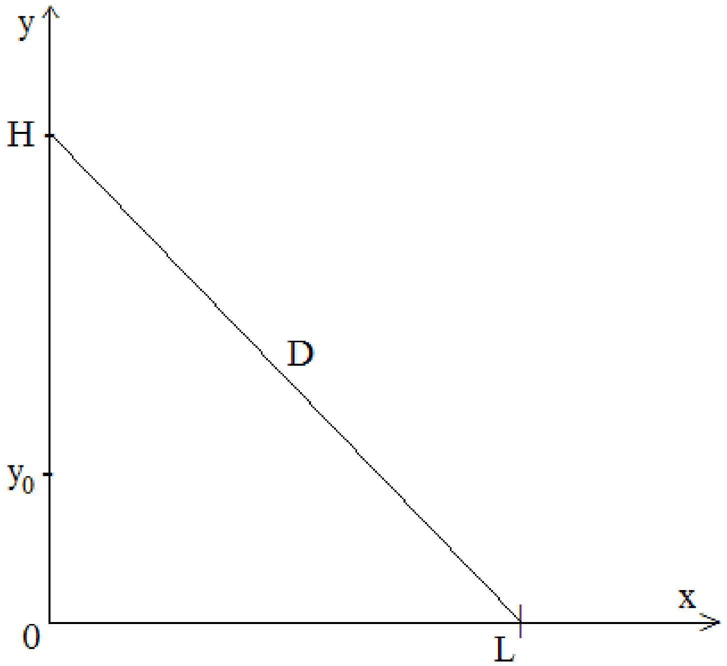
$$\frac{dx}{dt} = \frac{1}{8} \cdot \frac{16}{25} \cdot 2 = \frac{4}{25} = 0.16 \frac{rad}{s}.$$

Answer: (a) $1.2\ \frac{m}{s}$; **(b)** $0.16 \frac{rad}{s}$.

Question

6. A balloon is rising at a constant speed 4m/sec. A boy is cycling along a straight road at a speed of 8m/sec. When he passes under the balloon, it is 36 metres above him. How fast is the distance between the boy and balloon increasing 3 seconds later?

Solution



D is the distance between the boy and balloon, x is the (horizontal) distance traveled by the boy from the moment it passed under the balloon and y is the altitude of the balloon.

Applying the Pythagorean theorem to a right triangle

$$D^2 = x^2 + y^2$$

We know that

$$\mathrm{v}_x = \frac{dx}{dt} = 8 \frac{m}{sec}, \mathrm{v}_y = \frac{dy}{dt} = 4 \frac{m}{sec}, x = \mathrm{v}_x t, y = y_0 + \mathrm{v}_y t, y_0 = 36\,m.$$

Then

$$D^2 = (\mathrm{v}_x t)^2 + (y_0 + \mathrm{v}_y t)^2$$

Taking derivatives (with respect to time, t) on both sides we get

$$2D \frac{dD}{dt} = 2\mathrm{v}_x^2 t + 2\mathrm{v}_y (y_0 + \mathrm{v}_y t),$$

so that

$$\frac{dD}{dt} = \frac{1}{D} \cdot \left(\mathrm{v}_x^2 t + \mathrm{v}_y\left(y_0 + \mathrm{v}_y t\right)\right)$$

We have that

$$t = 3\ sec, D^2 = (8 \cdot 3)^2 + (36 + 4 \cdot 3)^2 = 2880\ (m^2), D = 24\sqrt{5}\ m\ .$$

Therefore

$$\frac{dD}{dt} = \frac{1}{24\sqrt{5}}\left(8^2 \cdot 3 + 4(36 + 4 \cdot 3)\right) = \frac{16}{\sqrt{5}} = \frac{16\sqrt{5}}{5}\ \left(\frac{m}{sec}\right).$$

Answer: $\frac{16\sqrt{5}}{5}\frac{m}{sec}$.