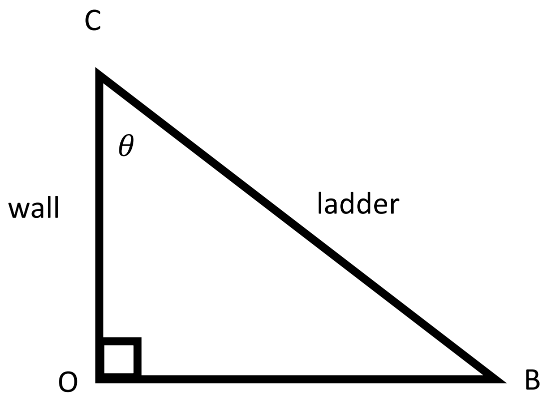
Answer on Question #64598 – Math – Calculus

Question

1. A ladder $15\,ft$ long rests against a vertical wall. Its top slides down the wall while its bottom moves away along the level ground at a speed of $2\,ft/s$. How fast is the angle between the top of the ladder and the wall changing when the angle is $\pi/3$ radians?

Solution

1. Ladder and wall can be regarded as sides of a right triangle.



At $t = 0$, $OB = 0$ and $OC \equiv BC = 15\,ft$. $OB = 2t\ [ft]$

$$\sin\theta = \frac{OB}{BC} \rightarrow \theta = \operatorname{asin}\left(\frac{OB}{BC}\right)$$

$$\theta = \operatorname{asin}\left(\frac{2t}{15}\right) \rightarrow d\theta = \frac{2}{\sqrt{225-4t^2}}dt \rightarrow \frac{d\theta}{dt} = \frac{2}{\sqrt{225-4t^2}}$$

Determine t, when $\theta = \frac{\pi}{3}$:

$$\sin\left(\frac{\pi}{3}\right) = \frac{2t_1}{15} \rightarrow \frac{\sqrt{3}}{2} = \frac{2t_1}{15} \rightarrow t_1 = \frac{15\sqrt{3}}{4}$$

Then,

$$\frac{d\theta}{dt}\Big|_{t=t_1} = \frac{2}{\sqrt{225-4t_1^2}} = \frac{2}{\sqrt{225-4\cdot\left(\frac{15\sqrt{3}}{4}\right)^2}}$$

$$\frac{d\theta}{dt}\Big|_{t=t_1} = \frac{2}{\sqrt{225-225\cdot\frac{3}{4}}} = \frac{2}{\sqrt{\frac{225}{4}}} = \frac{4}{\sqrt{225}} = \frac{4}{15}\left[\frac{rad}{s}\right]$$

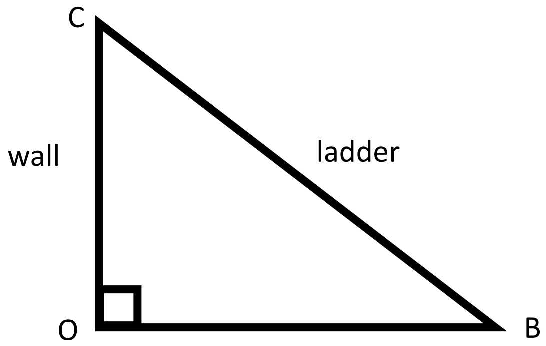
Answer: $\frac{4}{15}\left[\frac{rad}{s}\right]$,

Question

2. A ladder 12 meters long leans against a wall. The foot of the ladder is pulled away from the wall at the rate $12\, m/min$. At what rate is the top of the ladder falling when the foot of the ladder is 4 meters from the wall?

Solution

2. Similar to previous one.



At $t = 0$, $OB = 0$ and $OC = BC = 12\, m;\ OB = 12t\ (min)\ [m]$

By the Pythagorean theorem, $OC = \sqrt{BC^2 - OB^2}$.

Determine time t_1 when the foot of the ladder is 4 meters from the wall: $t_1 = \frac{4}{12} = \frac{1}{3}$

Ladder top is falling at rate:

$$\frac{d(OC)}{dt} = \frac{1}{2}\frac{1}{\sqrt{BC^2 - OB^2}}(-2 \cdot OB)\frac{d(OB)}{dt} = -12t \cdot 12 \cdot \frac{1}{\sqrt{12^2 - 12^2t^2}} = -\frac{12t}{\sqrt{1 - t^2}}$$

Plug in time t_1:

$$\frac{d(OC)}{dt}\bigg|_{t = t_1} = -\frac{12t_1}{\sqrt{1 - t_1^2}} = -\frac{12}{\sqrt{\frac{1}{t_1^2} - 1}} = -\frac{12}{\sqrt{9 - 1}} = -\frac{12}{2\sqrt{2}} = -3\sqrt{2}\ \left[\frac{m}{s}\right]$$

“minus” sign represents falling.

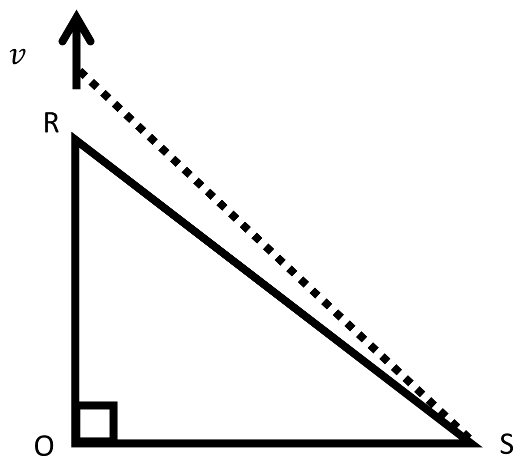
Answer: $-3\sqrt{2}\left[\frac{m}{s}\right]$,

Question

3. A rocket R is launched vertically and its tracked from a radar station S which is 4 miles away from the launch site at the same height above sea level. At a certain instant after launch, R is 5 miles away from S and the distance from R to S is increasing at a rate of 3600 miles per hour. Compute the vertical speed v of the rocket at this instant.

Solution

3.



By the Pythagorean theorem,

$$RS^2 = OR^2 + OS^2 \to OR = \sqrt{RS^2 - OS^2}$$

Let us find projection of speed v on RS:

$$v_{RS} = v\cos(\angle ORS) = v\frac{OR}{RS} = v\frac{\sqrt{RS^2 - OS^2}}{RS}$$

Then for the given instant we have

$$3600 = v\frac{\sqrt{5^2-4^2}}{5} \to 3600 = v\frac{3}{5} \to v = 6000\ \left[\frac{miles}{hour}\right].$$

Answer: $6000\left[\frac{miles}{hour}\right]$.