Answer on Question #64582 – Math – Differential Equations

Question

Find an equation of orthogonal trajectory of the curve of each of the following:

i)

$$x = Cy^2$$

Solution

Differentiating $x = Cy^2$ with respect to x

$$1 = 2Cy\frac{dy}{dx},$$

where $\frac{dy}{dx} = y'$.

Substituting $y' \rightarrow (-1/y')$

$$1 = 2Cy \cdot \frac{1}{-\frac{dy}{dx}}$$

$$\frac{dy}{dx} = -2Cy.$$

Integrating both sides

$$-2C\int dx = \int \frac{dy}{y},$$

$$-2Cx + k_1 = \ln|y|,$$

$$-2Cx + lnk_2 = ln|y|,$$

$$-2Cx = ln\frac{|y|}{k_2},$$

$$e^{-2Cx} = \frac{|y|}{k_2},$$

$$y = ke^{-2Cx}.$$

Answer: $y = ke^{-2Cx}$.

Question

Find an equation of orthogonal trajectory of the curve of each of the following:

ii)

$$x^2 + y^2 = Cx$$

Solution

Differentiating $x^2 + y^2 = Cx$ with respect to x:

$$2x + 2y\frac{dy}{dx} = C.$$

Substituting $y' \rightarrow (-1/y')$

$$2x - 2y\frac{dx}{dy} = C,$$

$$2x - C = 2y\frac{dx}{dy},$$

$$dy(2x - C) = 2ydx.$$

Integrating both sides

$$\int \frac{dy}{2y} = \int \frac{dx}{2x-C},$$

$$\frac{1}{2}\ln|2y| = \frac{1}{2}\ln|(2x - C)|,$$

$$y = x + k \text{ or } y = -x + k.$$

Answer: $y = x + k;\ y = -x + k.$

Question

Find an equation of orthogonal trajectory of the curve of each of the following:

iii)

$$y = e^{Cx}$$

Solution

Differentiating $y = e^{Cx}$ with respect to x

$$\frac{dy}{dx} = Ce^{Cx}$$

Substituting $y' \rightarrow (-1/y')$

$$\frac{\frac{-1}{dy}}{\frac{dx}{dx}} = Ce^{Cx},$$

$$-\frac{dx}{dy} = Ce^{Cx},$$

$$-e^{-Cx}dx = Cdy.$$

Integrating both sides

$$-\int e^{-Cx}dx = C\int dy,$$

$$\frac{1}{C}e^{-Cx} + k_1 = Cy,$$

$$e^{-Cx} + k_2 = C^2y,$$

$$y = \frac{e^{-Cx}}{C^2} + k.$$

Answer: $y = \frac{e^{-Cx}}{C^2} + k.$

Question

Find an equation of orthogonal trajectory of the curve of each of the following:

iv)

$$xy = C$$

Solution

Differentiating $xy = C$ with respect to x

$$y + x\frac{dy}{dx} = 0$$

Substitute $y' \rightarrow (-1/y')$

$$y + x \cdot \frac{-1}{\frac{dy}{dx}} = 0,$$

$$y - x\frac{dx}{dy} = 0,$$

$$xdx = ydy.$$

Integrating both sides

$$\int ydy = \int xdx,$$

$$\frac{y^2}{2} = \frac{x^2}{2} + C_1,$$

$$y^2 - x^2 = k.$$

Answer: $y^2 - x^2 = k.$

Question

Find an equation of orthogonal trajectory of the curve of each of the following:

v)

$$y^2 = x^2 + Cx$$

Solution

Differentiating $y^2 = x^2 + Cx$ with respect to x

$$2y\frac{dy}{dx} = 2x + C.$$

Substitute $y' \rightarrow (-1/y')$

$$2y\frac{-1}{\frac{dy}{dx}} = 2x + C,$$

$$-2y\frac{dx}{dy} = 2x + C,$$

$$-2\frac{dx}{2x+C} = \frac{dy}{y}.$$

Integrating both sides

$$-2\int\frac{dx}{2x+C} = \int\frac{dy}{y},$$

$$-\ln|2x + C| + lnk_1 = ln|y|,$$

$$ln\frac{k_1}{|2x+C|} = ln|y|,$$

$$y = \frac{k}{2x+C}.$$

Answer: $y = \frac{k}{2x+C}$.