Answer on Question #64533 – Math – Real Analysis

Question

If $0 < a < b$, determine $\lim\limits_{n\to\infty} \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$.

Solution

$$\lim_{n\to\infty} \frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \lim_{n\to\infty} \frac{\frac{a^{n+1}+b^{n+1}}{b^n}}{\frac{a^n+b^n}{b^n}} = \lim_{n\to\infty} \frac{b+\frac{a^n}{b^n}a}{1+\frac{a^n}{b^n}} = \frac{\lim\limits_{n\to\infty}\left(b+\frac{a^n}{b^n}a\right)}{\lim\limits_{n\to\infty}\left(1+\frac{a^n}{b^n}\right)} = \frac{\lim\limits_{n\to\infty} b+\lim\limits_{n\to\infty}\frac{a^n}{b^n}a}{\lim\limits_{n\to\infty} 1+\lim\limits_{n\to\infty}\frac{a^n}{b^n}} =$$

$$= \frac{b+a\lim\limits_{n\to\infty}\left(\frac{a}{b}\right)^n}{1+\lim\limits_{n\to\infty}\left(\frac{a}{b}\right)^n} = \frac{b+a\cdot 0}{1+0} = b.$$

If $0 < a < b$, then $\frac{a}{b} < 1$ and $\lim\limits_{n\to\infty}\left(\frac{a}{b}\right)^n = 0$.

Answer: b.