Answer on Question #64530 – Math – Real Analysis

Question

Determine the following limits:
(a) lim((3√n)1/2n), **(b)** lim((n+1)1/ln(n+1))

Solution

(a) $\lim\limits_{n\to\infty}\frac{3\sqrt{n}}{2n}=\frac{3}{2}\lim\limits_{n\to\infty}\frac{1}{\sqrt{n}}=0$

(b) We use the Stolz-Cesàro theorem:

$$\lim_{n\to\infty}\frac{a_n}{b_n}=\lim_{n\to\infty}\frac{a_n-a_{n-1}}{b_n-b_{n-1}},$$

where $a_n=n+1$, the sequence $b_n=\ln(n+1)$ is strictly monotone (strictly increasing) and divergent (approaches $+\infty$).

So we have

$$\lim_{n\to\infty}\frac{n+1}{\ln(n+1)}=\lim_{n\to\infty}\frac{(n+1)-n}{\ln(n+1)-\ln n}=\lim_{n\to\infty}\frac{1}{\ln\left(\frac{n+1}{n}\right)}=\frac{1}{\ln\left(\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)\right)}=1/ln1=\infty.$$

Answer: (a) 0; **(b)** ∞.