Answer on Question #64527 – Math – Real Analysis

Question

If (bn) is a bounded sequences and lim(an)=0, show that lim(anbn)=0 (explain why the theorem 3.2.3 from book real analysis 3rd edition, by Robert G Bartle can not be used)

Solution

The sequencer $\{b_n\}$ is bounded, that means:

$$\exists C < \infty, \quad \forall n \in \mathbb{N}: \ |b_n| \le C$$

If $\lim_{n\to\infty} a_n = 0$, then

$$\forall \varepsilon_1 > 0, \qquad \exists N \in \mathbb{N}, \quad \forall n \ge N: |a_n| < \varepsilon_1$$

Using this we can write

$$\forall \varepsilon > 0, \ \exists N \in \mathbb{N}, \ \forall n \ge N: \ \ |a_n||b_n| = |a_n b_n| < \varepsilon_1 C = \varepsilon,$$

where we define new ε as $\varepsilon = \varepsilon_1 C$.

By definition that means

$$\lim_{n\to\infty} a_n b_n = 0 \text{ by definition.}$$

The theorem 3.2.3 cannot be used here, because in this theorem two convergent sequences are used. In our case the sequence $\{b_n\}$ is bounded, it can be divergent: for example, $b_n = (-1)^n$.

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