Answer on Question #64526 – Math – Real Analysis

Question

Find the limit of the following sequences:

(a) $\lim\left(2+\frac{1}{n^2}\right)$, **(b)**$\lim\frac{(-1)^n}{n+2}$,

(**c**) $\lim\frac{\sqrt{n}-1}{\sqrt{n}+1}$

(d) $\lim\frac{n+1}{n\sqrt{n}}$

Solution

(a) $\lim\limits_{n\to\infty}\left(2+\frac{1}{n^2}\right)=\lim\limits_{n\to\infty}2+\lim\limits_{n\to\infty}\frac{1}{n^2}=2+\lim\limits_{n\to\infty}\frac{1}{n^2}$.

So if $\varepsilon>0$ is given, then choosing $N>\frac{1}{\sqrt{\varepsilon}}$ we have $\frac{1}{n^2}<\varepsilon,\ n>N$.

Thus $\lim\limits_{n\to\infty}\frac{1}{n^2}=0$ and therefore $\lim\limits_{n\to\infty}\left(2+\frac{1}{n^2}\right)=2$.

(b) $\lim\limits_{n\to\infty}\frac{(-1)^n}{n+2}$.

$$-\frac{1}{n+2}\le\frac{(-1)^n}{n+2}\le\frac{1}{n+2}.$$

Since $\lim\limits_{n\to\infty}\frac{-1}{n+2}=\lim\limits_{n\to\infty}\frac{1}{n+2}=0$, by Squeeze Theorem, $\lim\limits_{n\to\infty}\frac{(-1)^n}{n+2}=0$.

(c) $\lim\limits_{n\to\infty}\frac{\sqrt{n}-1}{\sqrt{n}+1}$.

$$\left|\frac{\sqrt{n}-1}{\sqrt{n}+1}-1\right|=\left|\frac{\sqrt{n}-1-\left(\sqrt{n}+1\right)}{\sqrt{n}+1}\right|=\frac{2}{\sqrt{n}+1}$$

If $\varepsilon>0$ is given, then choosing $N>1+\left(\frac{2}{\varepsilon}\right)^2$ we have $\left|\frac{\sqrt{n}-1}{\sqrt{n}+1}-1\right|<\varepsilon,\ n>N$.

Thus $\lim\limits_{n\to\infty}\frac{\sqrt{n}-1}{\sqrt{n}+1}=1$.

(d) $\lim\limits_{n\to\infty}\frac{n+1}{n\sqrt{n}}$.

$$\frac{n+1}{n\sqrt{n}}=\frac{1}{\sqrt{n}}+\frac{1}{n\sqrt{n}}$$

If $\varepsilon>0$ is given, then choosing $N>\frac{1}{\varepsilon^2}$ we have $\frac{1}{\sqrt{n}}<\varepsilon,\ n>N$.

Hence

$$\lim_{n\to\infty}\frac{1}{\sqrt{n}}=0.$$

It also true that

$$\lim_{n\to\infty}\frac{1}{n\sqrt{n}}=0.$$

Thus,

$$\lim_{n\to\infty}\frac{n+1}{n\sqrt{n}}=0.$$

Answer: (a) 2; **(b)** 0; **(c)** 1; **(d)** 0.