Answer on Question #64498 – Math – Calculus

Question

Determine the value of a, b, c and d so that the curve $y = ax^3 + bx^2 + cx + d$ will pass through the points (0,1), (-3,7) and have a critical point at (-1,3).

Solution

To determine the values of $a,\ b,\ c$ and d we shall substitute coordinates of the given points into the equation of the curve

$$y = ax^3 + bx^2 + cx + d.$$

At the point (0,1) compute

$$y(0) = 0 + 0 + 0 + d = 1.$$

Then

$$d = 1.$$

At the point (-3,7) compute

$$y(-3) = a(-3)^3 + b(-3)^2 + c(-3) + 1 = -27a + 9b - 3c + 1 = 7,$$

hence

$$-27a + 9b - 3c = 6.$$

At the point (-1,3) compute

$$y(-1) = a(-1)^3 + b(-1)^2 + c(-1) + 1 = -a + b - c + 1 = 3.$$

The point (-1,3) is also the critical point, hence the derivative at $x = -1$ is zero. The derivative is given by

$$y' = 3ax^2 + 2bx + c.$$

Set the derivative at $x = -1$ to zero:

$$y'(-1) = 3a(-1)^2 + 2b(-1) + c = 3a - 2b + c = 0.$$

To determine $a,\ b,\ c$ we solve the following system of equations:

$$\begin{cases} -27a + 9b - 3c = 6, \\ -a + b - c = 2, \\ 3a - 2b + c = 0. \end{cases}$$

Dividing the first equation by 3, the second equation by (-1) and interchanging positions of the equations

$$\begin{cases} a - b + c = -2, \\ 3a - 2b + c = 0, \\ 9a - 3b + c = -2. \end{cases}$$

By the Gaussian elimination method, replace the second equation by the second equation minus three times the first equation, and replace the third equation by the third equation minus nine times the first equation:

$$\begin{cases} a - b + c = -2, \\ \qquad b - 2c = 6, \\ \qquad 6b - 8c = 16. \end{cases}$$

Dividing the third equation by 2, we get

$$\begin{cases} a - b + c = -2, \\ b - 2c = 6, \\ 3b - 4c = 8. \end{cases}$$

Replace the third equation by the third equation minus three times the second equation:

$$\begin{cases} a - b + c = -2, \\ b - 2c = 6, \\ 2c = -10. \end{cases}$$

Dividing the third equation by 2 we get

$$c = -5.$$

Substituting for c into the second equation

$$b - 2 \cdot (-5) = 6,$$

it follows that

$$b = 6 + 2c = 6 + 2 \cdot (-5) = 6 - 10 = -4$$

Substituting for b and c into the first equation

$$a - (-4) - 5 = -2$$

it follows that

$$a = b - c - 2 = -4 - (-5) - 2 = -4 + 5 - 2 = -1$$

Then

$$a = -1, b = -4, c = -5, d = 1$$

and the curve is

$$y = -x^3 - 4x^2 - 5x + 1.$$

Answer: $a = -1, b = -4, c = -5, d = 1.$
Curve: $y = -x^3 - 4x^2 - 5x + 1.$