Find the no. of integral solutions of: xy=2x-y.

$$xy = 2x - y \to xy + y = 2x \to y(1 + x) = 2x \to y = \frac{2x}{1 + x}$$

The graph of that has vertical asymptote x=-1 and horizontal asymptote y=2.

We plot that graph:



The only possibility of y having an integral value when x has an integral value, is for integral values of x when y is at least 1 unit away from its horizontal asymptote y=2, and that is when

$$\left|\frac{2x}{1+x} - 2\right| \ge 1$$

By ordinary methods of college algebra, that has solution $[-3,-1)\cup(-1,1]$. So we only need to try x-values in that region which are -3, -2, 0, and 1. Substituting those in

$$y = \frac{2x}{1+x}$$

we find the only four integral solutions: (-3,3),(-2,4),(0,0), (1,1). So the number of integral solutions is 4.