Task 1. $\mathbf{a}, \mathbf{b}, \mathbf{c}$ *are three unit vectors such that* $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = 1/2\mathbf{b}$. *Find the angle* $\mathbf{a}$ *makes with* $\mathbf{b}$ *and* $\mathbf{c}$ *(*$\mathbf{b}$ *and* $\mathbf{c}$ *being non-parallel).*

Solution. Use the triple product expansion:

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c},$$

where $\mathbf{a} \cdot \mathbf{c}$ and $\mathbf{a} \cdot \mathbf{b}$ are the scalar products. Thus,

$$(\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c} = 1/2\mathbf{b},$$

which is equivalent to

$$(\mathbf{a} \cdot \mathbf{c} - 1/2)\mathbf{b} = (\mathbf{a} \cdot \mathbf{b})\mathbf{c}.$$

Since we are given that $\mathbf{b}$ and $\mathbf{c}$ are non-parallel, this equality implies

$$\mathbf{a} \cdot \mathbf{c} - 1/2 = \mathbf{a} \cdot \mathbf{b} = 0.$$

Thus, $\mathbf{a} \cdot \mathbf{c} = 1/2$ and $\mathbf{a} \cdot \mathbf{b} = 0$. The second equality means that $\mathbf{a}$ and $\mathbf{b}$ are perpendicular. The first one means that the cosine of the angle between $\mathbf{a}$ and $\mathbf{c}$ equals $1/2$, because all the vectors have the unit length, and, therefore, this angle equals 60 degrees.
Answer. $\mathbf{a}$ makes 90 degrees with $\mathbf{b}$ and 60 degrees with $\mathbf{c}$.

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