Answer on Question #64355 – Math – Algebra

Question

Which of the following is a root of the polynomial shown below?

$$f(x) = x^3 + 2x^2 - x - 2$$

A: 2
B: 0
C: 3
D: 1

Solution

Method 1

Try to find at least one root.
Calculate

$$f(1) = 1^3 + 2 \cdot 1^2 - 1 - 2 = 1 + 2 - 1 - 2 = 0.$$

It means that 1 is a root of $f(x) = x^3 + 2x^2 - x - 2$.
Search for other roots.
Divide $f(x)$ by $(x - 1)$

1	x^3 1	x^2 2	x^1 -1	x^0 -2
	1	$2 + 1 \cdot 1 = 3$	$-1 + 1 \cdot 3 = 2$	$-2 + 1 \cdot 2 = 0$
	1	3	2	
	$x^2 + 3x^1 + 2$			

Therefore

$$f(x) = x^3 + 2x^2 - x - 2 = (x - 1)(x^2 + 3x + 2).$$

Solving
$x^2 + 3x + 2 = 0,$
$D = 3^2 - 4 \cdot 1 \cdot 2 = 9 - 8 = 1,$
$x = \frac{-3-1}{2} = \frac{-4}{2} = -2$ or $x = \frac{-3+1}{2} = \frac{-2}{2} = -1.$
Then $x^2 + 3x + 2 = (x + 2)(x + 1).$
Therefore
$f(x) = x^3 + 2x^2 - x - 2 = (x - 1)(x^2 + 3x + 2) = (x - 1)(x + 1)(x + 2) = 0,$
hence

$$x - 1 = 0 \text{ or } x + 1 = 0 \text{ or } x + 2 = 0.$$

Thus, $x_1 = 1, x_2 = -1, x_3 = -2.$
The roots of $f(x)$ are $-2, -1, 1.$
The correct answer is "D: 1".
Answer: D: 1.

Method 2

Recall that

$$a^2 - b^2 = (a - b)(a + b),$$

so

$$f(x) = x^3 + 2x^2 - x - 2 = (x^3 - x) + (2x^2 - 2) = x(x^2 - 1) + 2(x^2 - 1) =$$

$= (x^2 - 1)(x + 2) = (x - 1)(x + 1)(x + 2) = 0$, hence

$x - 1 = 0$ or $x + 1 = 0$ or $x + 2 = 0$.

Thus, $x_1 = 1$, $x_2 = -1$, $x_3 = -2$

The roots of $f(x)$ are $-2, -1, 1$.

The correct answer is "D: 1".

Answer: D: 1.

Method 3

Calculate

$f(2) = 2^3 + 2 \cdot 2^2 - 2 - 2 = 8 + 8 - 2 - 2 = 12 \neq 0,$

$f(0) = 0^3 + 2 \cdot 0^2 - 0 - 2 = 0 + 0 - 0 - 2 = -2 \neq 0,$

$f(3) = 3^3 + 2 \cdot 3^2 - 3 - 2 = 27 + 18 - 3 - 2 = 40 \neq 0,$

$f(1) = 1^3 + 2 \cdot 1^2 - 1 - 2 = 1 + 2 - 1 - 2 = 0.$

It means that among numbers 2, 0, 3, 1 only number 1 is a root of $f(x) = x^3 + 2x^2 - x - 2$.

The correct answer is "D: 1".

Answer: D: 1.