**Answer on Question #64237 – Math – Statistics and Probability**

A bank is interested in studying the number of people who use the ATM located outside its office late at night. On average, 1.6 customers used the ATM during any 10 minute interval between 9 pm and Midnight.

**Question**

**i.** What is the probability of exactly 3 customers using the ATM during any 10 minute interval?

**Solution**

Let $\xi$ be the number of people using the ATM during any 10 minute interval. It is known that $\xi$ has the Poisson distribution with rate $\lambda = 1.6$. So the required probability is

$$P(\xi = 3) = \frac{\lambda^3}{3!}e^{-\lambda} = \frac{1.6^3}{3!}e^{-1.6} = \frac{4.096}{6}e^{-1.6} \approx 0.1378.$$

**Answer:** 0.1378.

**Question**

**ii.** What is the probability of 3 or fewer customers using the ATM during any 20 minute interval?

**Solution**

Let $\eta$ be the number of people using the ATM during any 20 minute interval. If $X$ and $Y$ are independent, their distribution is Poisson with rates $\lambda_1$ and $\lambda_2$ respectively, then their sum $X + Y$ is distributed as a Poisson random variable with rate $\lambda_1 + \lambda_2$.

Obviously, on average $1.6 \cdot 2 = 3.2$ customers used the ATM during any 20 minute interval between 9 pm and Midnight. So $\eta$ has the Poisson distribution with rate $\lambda = 3.2$, and the required probability is

$$P(\eta \le 3) = P(\eta = 0) + P(\eta = 1) + P(\eta = 2) + P(\eta = 3) = \frac{(3.2)^0}{0!}e^{-3.2} + \frac{(3.2)^1}{1!}e^{-3.2}$$

$$+\frac{(3.2)^2}{2!}e^{-3.2} + \frac{(3.2)^3}{3!}e^{-3.2} = e^{-3.2}(1 + 3.2 + 5.12 + 5.4613) \approx 0.6025.$$

**Answer:** 0.6025.