Answer on Question #64235 – Math – Statistics and Probability

Question

If electricity power failures occur according to a Poisson distribution with an average of 3 failures every twenty weeks, calculate the probability that:

i. at most one failure during a particular week.

ii. exactly 4 failures within ten weeks.

Solution

i. Let the amount of the power failures during 1 week be L_1. The distribution of a random variable L_1 is a Poisson distribution with the average $\lambda_1 = \lambda$:

$$P(L_1 = l) = \frac{\lambda_1^l}{l!} e^{-\lambda_1}.$$

There are 3 failures during 20 weeks on average: $20E(L_1) = 3$, hence $E(L_1) = \lambda_1 = 3/20$ ($E(L_1)$ is the mathematical expectation of L_1, i.e., its

average).

Next,

$$P(L_1 \le 1) = P(L_1 = 0) + P(L_1 = 1) = \frac{\lambda_1^0}{0!} e^{-\lambda_1} + \frac{\lambda_1^1}{1!} e^{-\lambda_1} = \frac{(\lambda t)^0}{0!} e^{-\lambda t} + \frac{(\lambda t)^1}{1!} e^{-\lambda t} =$$

$$= e^{-\lambda t} + \lambda t e^{-\lambda t} = (1 + \lambda t)e^{-\lambda_1 t} = \left(1 + \frac{3}{20} \cdot 1\right) e^{-3/20} = \frac{23}{20} e^{-3/20} \approx 0.990.$$

ii. Let the amount of the power failures during 10 weeks be L_{10}. The distribution of a random variable L_{10} is a Poisson distribution with the average $\lambda_{10} = \lambda t$:

$$P(L_{10} = l) = \frac{\lambda_{10}^l}{l!} e^{-\lambda_{10}},$$

hence

$$P(L_{10} = 4) = \frac{(\lambda t)^4}{4!} e^{-\lambda t} = \frac{\left(\frac{3}{20} \cdot 10\right)^4}{4!} e^{-\frac{3}{20} \cdot 10} = \frac{\left(\frac{3}{2}\right)^4}{4!} e^{-\frac{3}{2}} = \frac{27}{128} e^{-\frac{3}{2}} \approx 0.047.$$

Indeed, the amount of the power failures during 20 weeks is a sum of 2 random variables with the mass function $\frac{\lambda_{10}^l}{l!} e^{-\lambda_{10}}$. Consequently, $2E(L_{10}) = 3$, $E(L_{10}) = \lambda_{10} = 3/2$.

Next,

$$P(L_{10} = 4) = \frac{\lambda_{10}^4}{4!} e^{-\lambda_{10}} = \frac{\left(\frac{3}{2}\right)^4}{4!} e^{-\frac{3}{2}} = \frac{27}{128} e^{-\frac{3}{2}} \approx 0.047$$

Answer:

i. The probability that at most one failure during a particular week is $\frac{23}{20} e^{-3/20} \approx 0.990$.

ii. The probability that exactly 4 failures within ten weeks is $\frac{27}{128} e^{-\frac{3}{2}} \approx 0.047$.