Answer on Question #64180 – Math – Statistics and Probability

Question

A dice is thrown 5 times . What is the probability of getting sum 25 ?

Solution

Method 1

Let A be the event of getting the sum of 25, N is the number of all possibilities and K is the number of possibilities satisfying A.

Let's use the classical definition of probability:

$$P(A) = \frac{K}{N}.$$

N is equal to the number of all possible variants to choose the outcome five times:

$$N = 6^5 = 7776.$$

We can receive K as the number of nonnegative integer solutions to the equation

$k_1 + k_2 + k_3 + k_4 + k_5 = 25$, where $1 \le k_i \le 6,\ i = 1, 2, 3, 4, 5$.

Let $k_1 = m_1 + 1,\ k_2 = m_2 + 1,\ k_3 = m_3 + 1,\ k_4 = m_4 + 1,\ k_5 = m_5 + 1,$ where $0 \le m_i \le 5, i = 1, 2, 3, 4, 5$.

Then

$k_1 + k_2 + k_3 + k_4 + k_5 = m_1 + 1 + m_2 + 1 + m_3 + 1 + m_4 + 1 + m_5 + 1 = 25,$

hence

$m_1 + m_2 + m_3 + m_4 + m_5 = 20,\ 0 \le m_i \le 5, i = 1, 2, 3, 4, 5.$

Let $P_1 \Leftrightarrow m_1 > 5 \Leftrightarrow m_1 \ge 6$, $P_2 \Leftrightarrow m_2 > 5 \Leftrightarrow m_2 \ge 6$, $P_3 \Leftrightarrow m_3 > 5 \Leftrightarrow m_3 \ge 6$, $P_4 \Leftrightarrow m_4 > 5 \Leftrightarrow m_4 \ge 6$, $P_5 \Leftrightarrow m_5 > 5 \Leftrightarrow m_5 \ge 6$.

There are $|S| = \binom{20 + 5 - 1}{5 - 1} = \binom{24}{4} = \frac{24 \cdot 23 \cdot 22 \cdot 21}{4 \cdot 3 \cdot 2} = 23 \cdot 22 \cdot 21 = 10626$ non-negative solutions to the equation $m_1 + m_2 + m_3 + m_4 + m_5 = 20, m_i \ge 0$.

Solutions in P_1 correspond to non-negative integer solutions of

$l_1 + m_2 + m_3 + m_4 + m_5 = 20 - 6$ after substitution $m_1 = l_1 + 6,\ l_1 \geq 0,$ the answer is $\binom{20-6+5-1}{5-1} = \binom{18}{4} = \frac{18\cdot17\cdot16\cdot15}{4\cdot3\cdot2} = 3\cdot17\cdot4\cdot15 = 3060.$

Solutions in $P_1 \cap P_2$ correspond to non-negative integer solutions of

$l_1 + l_2 + m_3 + m_4 + m_5 = 20 - 6 - 6$ after substitutions

$m_1 = l_1 + 6, m_2 = l_2 + 6, l_1 \geq 0, l_2 \geq 0$, the answer is

$\binom{20-6-6+5-1}{5-1} = \binom{12}{4} = \frac{12\cdot11\cdot10\cdot9}{4\cdot3\cdot2} = 11\cdot5\cdot9 = 495.$

Solutions in $P_1 \cap P_2 \cap P_3$ correspond to non-negative integer solutions of

$l_1 + l_2 + l_3 + m_4 + m_5 = 20 - 6 - 6 - 6$ after substitutions

$m_1 = l_1 + 6, m_2 = l_2 + 6, m_3 = l_3 + 6, l_1 \geq 0, l_2 \geq 0, l_3 \geq 0,$

the answer is $\binom{20-6-6-6+5-1}{5-1} = \binom{6}{4} = \frac{6\cdot5\cdot4!}{4!\cdot2!} = 15.$

Applying inclusion-exclusion formula, we have

$$|P_1 \cup P_2 \cup P_3 \cup P_4 \cup P_5| = \sum_{i=1}^{5}|P_i| - \sum_{1\leq i<j\leq5}|P_i \cap P_j| + \sum_{1\leq i<j<k\leq5}|P_i \cap P_j \cap P_k| =$$

$= 5\cdot3060 - 10\cdot495 + 10\cdot15 = 10500.$

We are interested in

$K = |\bar{P}_1 \cap \bar{P}_2 \cap \bar{P}_3 \cap \bar{P}_4 \cap \bar{P}_5| = |S| - |P_1 \cup P_2 \cup P_3 \cup P_4 \cup P_5| = 10626 -$
$-10500 = 126.$

Finally

$$P(A) = \frac{K}{N} = \frac{126}{7776}.$$

Method 2

We can receive K by writing down and counting all suitable situations sorted by minimal number received:

1) $k_1 = 1, k_2 = 6, k_3 = 6, k_4 = 6, k_5 = 6$. There are 5 ways depending on a number thrown.
2) $k_1 = 2, k_2 = 5, k_3 = 6, k_4 = 6, k_5 = 6$. There are 5 ways to locate “2” and 4 possibilities to get “5” at once. So we have $5\cdot4 = 20$ variants.

3)

A) $k_1 = 3, k_2 = 4, k_3 = 6, k_4 = 6, k_5 = 6$. (Similarly, 20 variants)

B) $k_1 = 3, k_2 = 5, k_3 = 5, k_4 = 6, k_5 = 6$. In this case there are 5 possibilities to locate “3”, 4 possibilities to get “5” for the first time and 3 possibilities to get “5” for the second time.

We get $5 \cdot 4 \cdot 3 = 60$ variants.

4) $k_1 = 4, k_2 = 5, k_3 = 5, k_4 = 5, k_5 = 6$. (20 variants)
5) $k_1 = 5, k_2 = 5, k_3 = 5, k_4 = 5, k_5 = 5$ (1 variant)

Variant of getting “6” five times is impossible, because the sum of numbers will be 30 then.

Now

$$K = 5 + 20 + 20 + 60 + 20 + 1 = 126,$$

Finally

$$P(A) = \frac{K}{N} = \frac{126}{7776}.$$

Answer: $\frac{126}{7776}$.