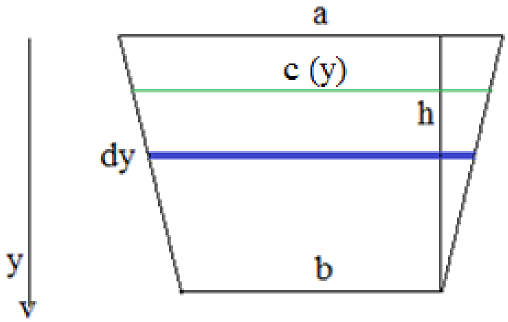
**Answer on Question #64141 – Math – Geometry**

**Question**

3. A trough having a trapezoidal cross section is full of water. If the trapezoid is 3ft wide at the top, 2ft wide at the bottom, and 2ft deep, find the total force owing to water pressure on one end of the trough.

**Solution**




$\Delta F = pressure \cdot Area = (w \cdot depth) \cdot (length \cdot width) = \left(w \cdot h(y)\right) \cdot (c(y) \cdot \Delta y)$

The force F exerted by a fluid of constant weight-density $w$ (per unit of volume) against a submerged vertical plane region from $y = y_1$ to $y = y_2$ is

$$F = w \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} h(y_i)\, c(y_i) \Delta y = w \int_{y_1}^{y_2} h(y) c(y)\, dy,$$

where $h(y)$ is the depth of the fluid at $y$ and $c(y)$ is the horizontal length of the region on $y$.

$S_{trap} = \frac{a+b}{2} h = \frac{a + c(y)}{2} y + \frac{c(y) + b}{2} (h - y);$

$ah + bh = ay + c(y)y + c(y)h + bh - c(y)y - by;$

$ah = ay + c(y)h - by;$

$c(y) = \frac{ah - ay + by}{h} = a - \frac{a-b}{h} y.$

$h(y) = y, c(y) = a - \frac{a-b}{h} y$

We have that

$w = 62.4\ lb/ft^3, a = 3\ ft, b = 2\ ft, h = 2\ ft.$

$F = w \int_0^h y \left(a - \frac{a-b}{h} y\right) dy;$

$$F = 62.4 \int_0^2 y \left(3 - \frac{3-2}{2} y\right) dy = 62.4 \left(\frac{3}{2} y^2 - \frac{1}{6} y^3\right) \Big|_0^2 = 62.4 \left(\frac{3}{2} \cdot 2^2 - \frac{1}{6} \cdot 2^3\right)$$

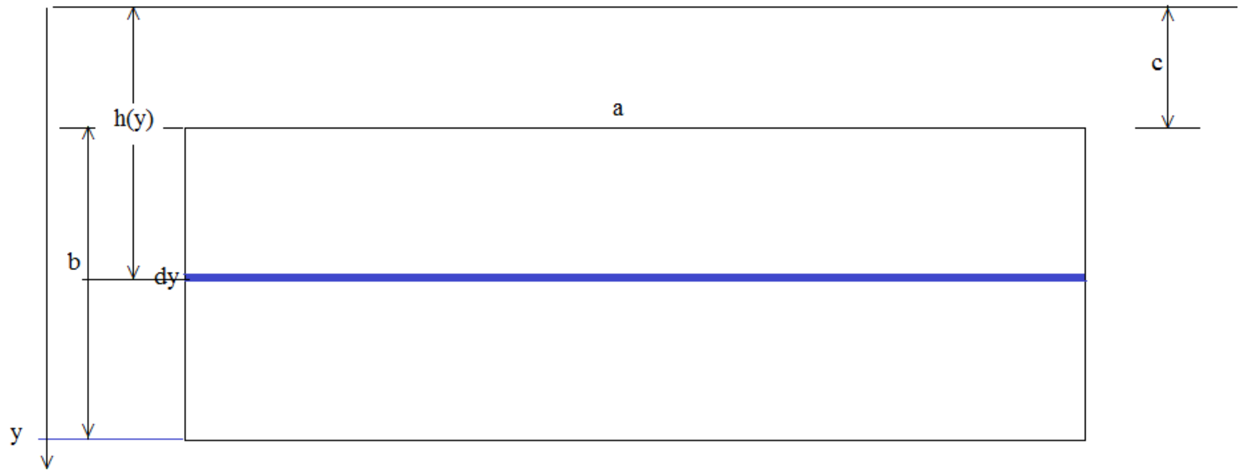
$$= 62.4 \left(6 - \frac{4}{3}\right) = 62.4 \cdot \frac{14}{3} = 291.2\ (lb).$$

**Answer:** 291.2 lb.

**Question**

4. Find the total force on the dam due to the fluid pressure:
Rectangle 200ft wide, 15ft high; water 10ft deep

**Solution**




$\Delta F = pressure \cdot Area = (w \cdot depth) \cdot (length \cdot width) = \left(w \cdot h(y)\right) \cdot (c(y) \cdot \Delta y)$

The force $F$ exerted by a fluid of constant weight-density $w$ (per unit of volume) against a submerged vertical plane region from $y = y_1$ to $y = y_2$ is

$$F = w \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} h(y_i)\, c(y_i) \Delta y = w \int_{y_1}^{y_2} h(y) c(y)\, dy,$$

where $h(y)$ is the depth of the fluid at $y$ and $c(y)$ is the horizontal length of the region on $y$:

$h(y) = y, c(y) = a.$

We have that

$w = 62.4\ lb/ft^3, a = 200\ ft, b = 15\ ft, c = 10\ ft.$

$$F = w \int_{c}^{c+b} y \cdot a\, dy;$$

$$F = 62.4 \int_{10}^{10+15} y200\, dy = 62.4 \cdot 200 \frac{y^2}{2} \Big|_{10}^{25} = 62.4 \cdot 100 \cdot (25^2 - 10^2) =$$

$$= 3276000\ (lb).$$

**Answer:** 3276000 lb.

**Answer provided by** www.AsignmenmtExpert.com