Question#6411. The line of shortest distance of the lines (x/2)=(-y/3)=(z) and (x-2)/3=(y-1)/-5=(z+2)/2 intersects these two lines in P andQ respectively.

Find the shortest distance between these two lines, find the coordinates of P and of Q.

Solution

$$l_1 : \frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$$

$$l_2 : \frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$$

The shortest distance between these two lines we will find by the formula:

$$d = \frac{|M_1M_2 * \vec{a}|}{|\vec{a}|}$$

$$\vec{a} = \vec{s}_1 \times \vec{s}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & 1 \\ 3 & -5 & 2 \end{vmatrix} = -\vec{i} - \vec{j} - \vec{k};$$

$$M_1M_2 = (-2-1;-1-0;2-0) = (-3;-1;2)$$

$$|\vec{a}| = \sqrt{3}$$

$$|M_1M_2 * \vec{a}| = (-3)*(-1) + (-1)*(-1) + (2)*(-1) = 2$$

$d = \frac{|M_1M_2 * \vec{a}|}{|\vec{a}|} = \frac{2}{\sqrt{3}}$ - the shortest distance between the lines l_1 and l_2

The canonical equation of the line, which is the shortest distance between the lines l_1 and l_2 :

$\frac{x-a}{m} = \frac{y-b}{n} = \frac{z-c}{k}$, where $\vec{a} = (m;n;k) = (-1;-1;-1)$

$$\frac{x-a}{-1} = \frac{y-b}{-1} = \frac{z-c}{-1}$$