

Answer on Question #63977 – Math – Trigonometry

Question

Find the value of

$$\sin 4A - \cos 4A = \sin 2A - \cos 2A$$

Solution

Using formulae

$$\sin x - \sin y = 2 \sin \frac{x-y}{2} \cdot \cos \frac{x+y}{2};$$

$$\cos x - \cos y = -2 \sin \frac{x+y}{2} \cdot \sin \frac{x-y}{2};$$

compute

$$\sin 4A - \cos 4A = \sin 2A - \cos 2A;$$

$$\sin 4A - \sin 2A = \cos 4A - \cos 2A;$$

$$2\sin A \cos 3A = -2\sin 3A \cdot \sin A;$$

$$\cos 3A = -\sin 3A \text{ or } \sin A = 0.$$

If $\cos 3A = -\sin 3A$, then

$$\cos 3A + \sin 3A = 0;$$

$$\sqrt{2} \sin(3A + \pi/4) = 0;$$

$$\sin(3A + \pi/4) = 0;$$

$$3A + \pi/4 = k\pi, k \in \mathbb{Z}$$

$$A = -\pi/12 + k\pi/3, k \in \mathbb{Z}.$$

If $\sin A = 0$, then $A = m\pi, m \in \mathbb{Z}$.

Thus, $A = -\pi/12 + k\pi/3, k \in \mathbb{Z}$ or $A = m\pi, m \in \mathbb{Z}$.

Answer: $A = -\pi/12 + k\pi/3, k \in \mathbb{Z}$; $A = m\pi, m \in \mathbb{Z}$.