Answer on Question #63917 – Math – Discrete Mathematics

Question

Let a relation $R = (x, -x) | x^2 = x \ (mod 2x)$, whereby x element of positive integer. Determine whether R is reflexive, symmetric, anti-symmetric or transitive.

Solution

Here $x^2 = x(mod2x)$ and $(x, -x) \in R$.

If $x^2 = x(mod2x)$, then $x^2 = x + 2kx = (2k + 1)x$,

$x = 2k + 1$.

1) *Is R reflexive?*

By a definition of a reflexive relation, all the pairs of (x, x) must be in R.

But only the pairs $(x, -x)$ exist in R.

The only possible case is $x = 0$, but 0 is not a positive number.

Thus, a relation <u>R is not reflexive.</u>

2) *Is R symmetric?*

By a definition of a symmetric relation, if (x, y) are in R, then (y, x) are in R too.

By a definition of R, only pairs $(x, -x)$ are in R, where x is a positive integer.

Pairs $(-x, x)$ may be regarded only when $x = 0$, but 0 is not a positive number.

Thus, a relation <u>R is not symmetric.</u>

3) Is *R anti-symmetric?*

By a definition of an anti-symmetric relation, if (x, y) are in R and (y, x) are in R, then $x = y$.

If $(x, -x)$ are in R and $(-x, x)$ are in R, then $x = -x$, that is, $x = 0$.

The conclusion is true in this problem.

Therefore, a relation R is anti-symmetric.

4) *Is R transitive?*

By a definition of a transitive relation, the following property holds true:

$$(x, y), (y, z) \in R \rightarrow (x, z) \in R.$$

By a definition of R, only pairs $(x, -x), (y, -y)$ are in R.

Using the two previous definitions one conclude

$-x = y$ and $-y = z \leftrightarrow -x = -z \leftrightarrow x = z.$

Thus, a relation R is transitive.

Answer:

1) nonreflexive; **2)** nonsymmetric; **3)** anti-symmetric; **4)** transitive.