Task 1. *Let p and q be two distinct primes. How many kinds of abelian groups are there of order p^3q (up to isomorphism)?*

Solution. By the fundamental theorem of finite abelian groups each finite abelian group is isomorphic to the direct sum of cyclic groups of prime-power order. In our case the following pairwise non-isomorphic decompositions are possible:

$$\begin{aligned}\mathbb{Z}_p \oplus \mathbb{Z}_p \oplus \mathbb{Z}_p \oplus \mathbb{Z}_q &\cong \mathbb{Z}_p \oplus \mathbb{Z}_p \oplus \mathbb{Z}_{pq};\\ \mathbb{Z}_p \oplus \mathbb{Z}_{p^2} \oplus \mathbb{Z}_q &\cong \mathbb{Z}_p \oplus \mathbb{Z}_{p^2q};\\ \mathbb{Z}_{p^3} \oplus \mathbb{Z}_q &\cong \mathbb{Z}_{p^3q}.\end{aligned}$$

Thus, there are 3 kinds of abelian groups of order p^3q.
Answer: 3. □