Answer on Question #63808 – Math – Algebra

Question

If you are given $y = 2\cos 3x$. What do you do? I mean how do you change your graph from base graph? What do you do with the 2 and 3?

Solution

Operations applied to the 'outside' of a function affect the vertical aspects of the graph. Operations applied to the 'inside' (argument) of a function affect the horizontal aspects of the graph.

We can see that the transformation $y = 2\cos(x)$ has a vertical effect on the graph of

$y = \cos(x)$, because it stretches $y = \cos(x)$ by 2 in the y-direction: the x coordinates stay the same and the y coordinates are multiplied by 2. Remember that the x-intercepts do not move under vertical stretches and shrinks: if $f(x) = 0$ for some value of x, then $kf(x) = 0$ for the same values of x. The x-intercepts of $y = \cos(x)$ are $x = -\frac{\pi}{2} + n\pi$, where n is integer.

Nevertheless, the transformation $y = \cos(3x)$ has a horizontal effect on the graph of

$y = \cos(x)$, because it shrinks (compresses) $y = \cos(x)$ by 3 in the x-direction:

the y coordinates stay the same and the x coordinates are divided by 3.

Though in most instances, we take the operations 'from the inside out', remember that the vertically-oriented transformations do not affect the horizontally-oriented transformations, and vice versa.

Therefore, the order of transformations does not matter in this problem and there exist two ways of the solution.

Method 1

If we want to obtain the graph of $y = 2\cos 3x$ from the base graph of $y = \cos x$,

we must do the following steps:

1) Vertically stretch the base graph by a factor of 2:

$$f(x) = \cos(x) \to g(x) = 2\cos(x).$$

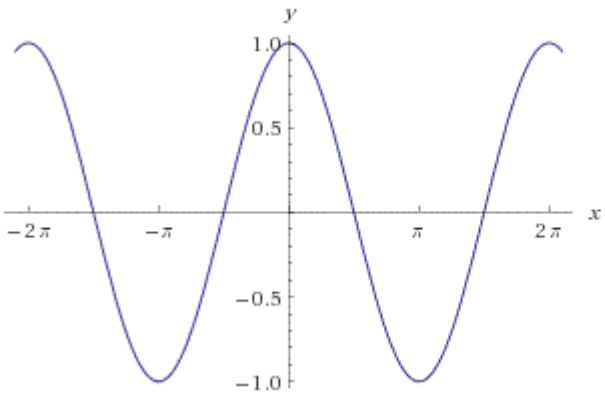

Fig. 1 Plot of $y = \cos(x)$

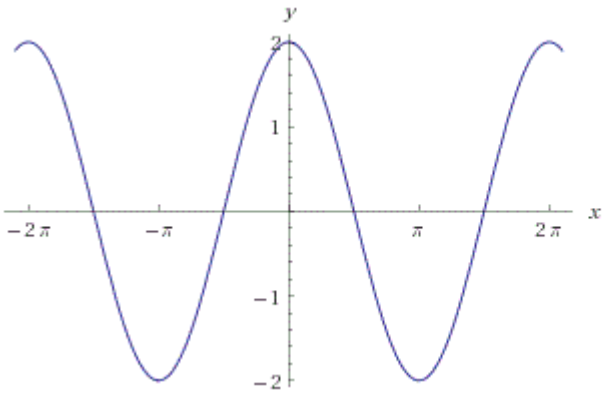

Fig. 2 Plot of $y = 2\cos(x)$

2) Horizontally shrink the graph by a factor of 3:

$$g(x) = 2\cos(x) \rightarrow h(x) = 2\cos(3x).$$

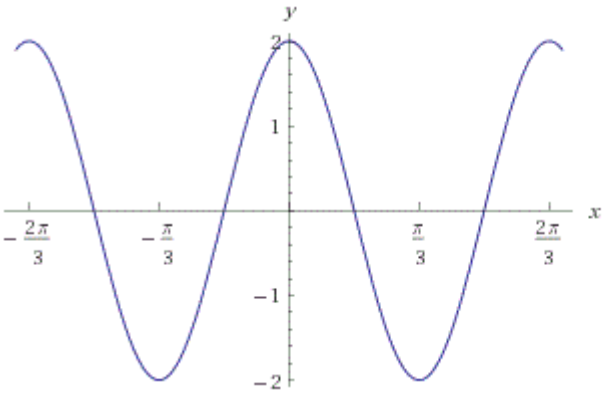

Fig. 3 Plot of $y = 2\cos(3x)$

Method 2

If we want to obtain the graph of $y = 2\cos 3x$ from the base graph of $y = \cos x$, we must do the following steps:

1) Horizontally shrink the base graph by a factor of 3:

$$p(x) = \cos(x) \rightarrow q(x) = \cos(3x).$$

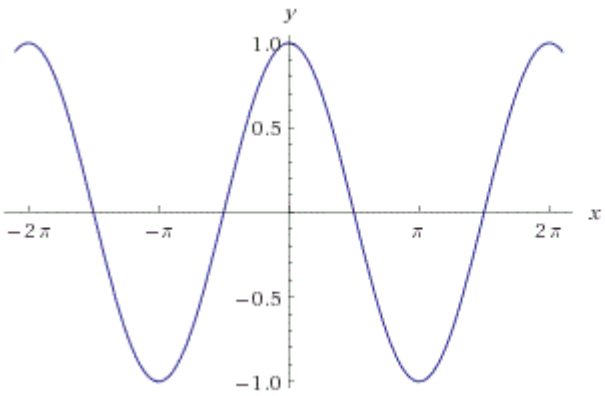

Fig. 4 Plot of $y = \cos(x)$

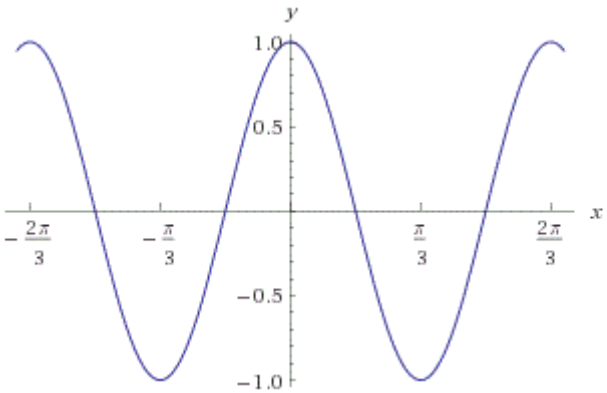

Fig. 5 Plot of $y = \cos(3x)$

2) Vertically stretch the graph by a factor of 2:

$$q(x) = \cos(3x) \rightarrow r(x) = 2\cos(3x).$$

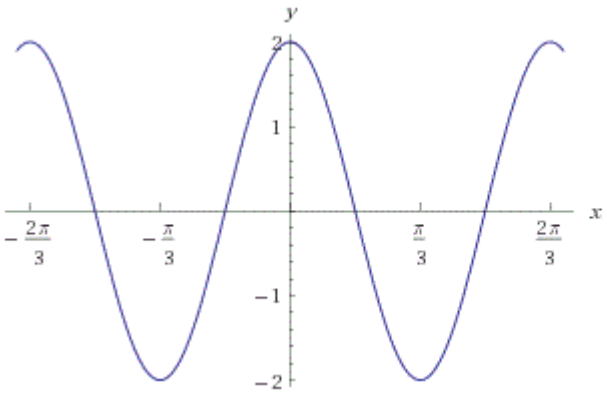

Fig. 6 Plot of $y = 2\cos(3x)$