Answer on Question #63631 – Math – Analytic Geometry

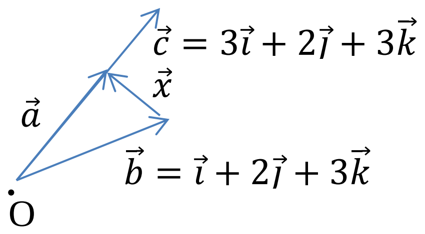
Question

Let O be the origin and OA = a1i +a2j + a3k. find the equation of a line that passes through the point (1 2 3) and is perpendicular to the vector 3i+2j+3k.

Solution

There are two ways of solution of this problem

1. The first method



$\vec{a} = m(3\vec{\imath} + 2\vec{\jmath} + 3\vec{k})$ is a projection of $\vec{b}$ on $\vec{c}$

$$\vec{b} = \vec{\imath} + 2\vec{\jmath} + 3\vec{k}$$

$$\vec{b} + \vec{x} = \vec{a}, \qquad \vec{x} = \vec{a} - \vec{b}$$

$$\vec{x} = (3m-1)\vec{\imath} + (2m-2)\vec{\jmath} + (3m-3)\vec{k}$$

Since $\vec{a} \perp \vec{x}$ then $\vec{a} \cdot \vec{x} = 0$ or

$$\left((3m-1)\vec{\imath} + (2m-2)\vec{\jmath} + (3m-3)\vec{k}\right) \cdot \left(3\vec{\imath} + 2\vec{\jmath} + 3\vec{k}\right) = 0$$

$$9m - 3 + 4m - 4 + 9m - 9 = 0$$

$$22m = 16$$

$$m = \frac{8}{11}$$

$$\vec{x} = \left(\frac{24}{11} - 1\right)\vec{\imath} + \left(\frac{16}{11} - 2\right)\vec{\jmath} + \left(\frac{24}{11} - 3\right)\vec{k}$$

Thus, the direction vector of the line is

$$\vec{n} = \frac{13}{11}\vec{\imath} - \frac{6}{11}\vec{\jmath} - \frac{9}{11}\vec{k}\ .$$

One more collinear vector is $11\left(\frac{13}{11}, -\frac{6}{11}, -\frac{9}{11}\right) = (13, -6, -9)$.

The line passes through the point (1 2 3).

Thus, the equation of the line is

$$\frac{x-1}{13} = \frac{y-2}{-6} = \frac{z-3}{-9}$$

Answer: $\frac{x-1}{13} = \frac{y-2}{-6} = \frac{z-3}{-9}$

2. The second method

$\vec{n}$ is a normal to the plane, where the vectors $\vec{a}$ and $\vec{b}$ lie,

$$\vec{n} = \vec{a}\times\vec{b} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 3 & 2 & 3 \\ 1 & 2 & 3 \end{vmatrix} = (2\cdot 3 - 2\cdot 3)\vec{\imath} - (3\cdot 3 - 1\cdot 3)\vec{\jmath} + (3\cdot 2 - 1\cdot 2)\vec{k} =$$

$$= (6-6)\vec{\imath} - (9-3)\vec{\jmath} + (6-2)\vec{k} = 0\vec{\imath} - 6\vec{\jmath} + 4\vec{k}$$

$a\times n$ is a vector that lies on this plane and is perpendicular to $\vec{a}$, and it is the direction vector of the line

$$a\times n = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 3 & 2 & 3 \\ 0 & -6 & 4 \end{vmatrix} = (8+18)\vec{\imath} - (12-0)\vec{\jmath} + (-18-0)\vec{k} = 26\vec{\imath} - 12\vec{\jmath} - 18\vec{k}$$

This is the direction vector of the line and one more collinear one is

$$\left(\frac{26}{2}, -\frac{12}{2}, -\frac{18}{2}\right) = (13, -6, -9).$$

The line passes through the point (1 2 3). Using the condition for collinearity of vectors we get that the equation of the line is

$$\frac{x-1}{13} = \frac{y-2}{-6} = \frac{z-3}{-9}$$

Answer: $\frac{x-1}{13} = \frac{y-2}{-6} = \frac{z-3}{-9}$.