Answer on Question #63630 – Math – Analytic Geometry

Question

Let O be the origin and $\overrightarrow{OA} = a_1\vec{\imath} + a_2\vec{\jmath} + a_3\vec{k}$. Find the equation of the planes such that every point on the plane is equidistant from the two ends of the vector $\overrightarrow{OA}$. You are asked to do this by first finding the point of intersection of the plane with the vector $\overrightarrow{OA}$. State the condition for vector $\overrightarrow{OA}$ to have a magnitude of 25.

Solution

Let $M(x, y, z)$ be a point in R^3. Then

$OM^2 = x^2 + y^2 + z^2$;

$AM^2 = (x - a_1)^2 + (y - a_2)^2 + (z - a_3)^2$.

Point M is equidistant from the two ends of the vector $\overrightarrow{OA}$.

$x^2 + y^2 + z^2 = (x - a_1)^2 + (y - a_2)^2 + (z - a_3)^2$;

$x^2 + y^2 + z^2 = x^2 + y^2 + z^2 - 2a_1x - 2a_2y - 2a_3z + a_1^2 + a_2^2 + a_3^2$;

$$a_1x + a_2y + a_3z - \frac{a_1^2 + a_2^2 + a_3^2}{2} = 0.$$

The result is a plane perpendicular to the vector $\overrightarrow{OA} = a_1\vec{\imath} + a_2\vec{\jmath} + a_3\vec{k}$.

The midpoint N of the vector $\overrightarrow{OA}$ is the point of intersection of the plane with the vector $\overrightarrow{OA}$.

Indeed, $N\left(\frac{a_1}{2}, \frac{a_2}{2}, \frac{a_3}{2}\right)$:

$$\frac{a_1^2}{2} + \frac{a_2^2}{2} + \frac{a_3^2}{2} - \frac{a_1^2 + a_2^2 + a_3^2}{2} \equiv 0.$$

If the vector $\overrightarrow{OA}$ has a magnitude of 25, then

$a_1^2 + a_2^2 + a_3^2 = 25^2$;

$a_1^2 + a_2^2 + a_3^2 = 625$.