Answer on Question #63035 – Math – Matrix | Tensor Analysis

Question

WHAT IS MEANT BY (0,2), (2,0) and (1,1) tensor?

Solution

The tensors are classified according to their type (n, m), where n is the number of contravariant indices, m is the number of covariant indices, and $n + m$ gives the total order of the tensor.

A mixed tensor of rank or order $(m + n)$

$$T^{i_1 i_2 \dots i_m}_{j_1 j_2 \dots j_n}$$

is contravariant of order m and covariant of order n if it obeys the transformation law

$$\bar{T}^{i_1 i_2 \dots i_m}_{j_1 j_2 \dots j_n} = \left[J\left(\frac{x}{\bar{x}}\right)\right]^W T^{a_1 a_2 \dots a_m}_{b_1 b_2 \dots b_n} \frac{\partial \bar{x}^{i_1}}{\partial x^{a_1}} \frac{\partial \bar{x}^{i_2}}{\partial x^{a_2}} \cdots \frac{\partial \bar{x}^{i_m}}{\partial x^{a_m}} \cdot \frac{\partial x^{b_1}}{\partial \bar{x}^{j_1}} \frac{\partial x^{b_2}}{\partial \bar{x}^{j_2}} \cdots \frac{\partial x^{b_n}}{\partial \bar{x}^{j_n}},$$

where

$$J\left(\frac{x}{\bar{x}}\right) = \left|\frac{\partial x}{\partial \bar{x}}\right| = \frac{\partial(x^1, x^2, \dots, x^N)}{\partial(\bar{x}^1, \bar{x}^2, \dots, \bar{x}^N)}$$

is the Jacobian of the transformation.

When $W = 0$ the tensor is called an absolute tensor, otherwise it is called a relative tensor of weight W.

A tensor field of type **(2,0)** on the n-dimensional smooth manifold M associates with each x a collection of n^2 smooth functions $T^{ij}(x^1, x^2, \dots, x^n)$ which satisfy the following transformation rule:

$$\bar{T}^{ij} = \frac{\partial \bar{x}^i}{\partial x^k} \frac{\partial \bar{x}^j}{\partial x^m} T^{km} \quad \text{('contravariant rank 2')}$$

Inverse metric tensor, bivector are examples of a (2,0)-tensor.

A tensor field of type **(1,1)** on the n-dimensional smooth manifold M associates with each x a collection of n^2 smooth functions $F^i_j(x^1, x^2, \dots, x^n)$ which satisfy the following transformation rule:

$$\bar{F}^i_j = \frac{\partial \bar{x}^i}{\partial x^k} \frac{\partial x_m}{\partial \bar{x}^j} F^k_m \quad \text{('mixed with contravariant rank 1 and covariant rank 1')}$$

A linear transformation is an example of a (1,1)-tensor.

A tensor field of type **(0,2)** on the n-dimensional smooth manifold M associates with each x a collection of n^2 smooth functions $E_{ij}(x^1, x^2, \dots, x^n)$ which satisfy the following transformation rule:

$$\bar{E}_{ij} = \frac{\partial x_k}{\partial \bar{x}^j}\frac{\partial x_m}{\partial \bar{x}^j} E_{km} \quad \text{('covariant rank 2')}$$

A bilinear form is an example of a (0,2)-tensor.