Answer on Question #63014 – Math – Differential Equations

Question

The decay of a population by catastrophic two body collisions is described by
dN / dt =- kN2.
where 2 is supersubscribe
derive the solution.
N(t)=No (1+ t/T) -1,where o is subscribe and -1
Supersubscribe

Solution

$$\frac{dN}{dt} = -kN^2$$

This is a separable differential equation. In order to solve it we have to separate the differential equation and integrate both sides.

$$\frac{dN}{N^2} = -kdt$$

$$-d\left(\frac{1}{N}\right) = -kdt$$

$$d\left(\frac{1}{N}\right) = kdt$$

Integration of both sides of the equation yields the general solution

$$\frac{1}{N} = k \cdot t + C,$$

where C is an integration constant.

Apply the initial condition and find the value of C:

when $t = 0$ we get

$$\frac{1}{N_0} = k \cdot 0 + C$$

$$\frac{1}{N_0} = C.$$

Plug C into the general solution.

$$\frac{1}{N} = k \cdot t + \frac{1}{N_0}$$

Solve for N

$$N = \frac{1}{kt + \frac{1}{N_0}}$$

$$N = \frac{1}{\frac{1}{N_0}(ktN_0 + 1)} = N_0(ktN_0 + 1)^{-1}$$

If we set $kN_0 = 1/T$, then

$$N = N_0\left(\frac{t}{T} + 1\right)^{-1}.$$

Answer: $N = N_0\left(\frac{t}{T} + 1\right)^{-1}$, where $T = \frac{1}{kN_0}$.