Answer on Question #62954 – Math – Differential Equations

Question

Solve this equation

$$\frac{d^2B_z}{dr^2} + \frac{1}{r}\frac{dB_z}{dr} - \frac{B_z}{a^2} = 0,$$

where a=(m/u ne²)½

Solution

$$\frac{d^2B_z}{dr^2} + \frac{1}{r}\frac{dB_z}{dr} - \frac{B_z}{a^2} = 0$$

Multiply equation by r^2 , we get

$$r^2\frac{d^2B_z}{dr^2} + r\frac{dB_z}{dr} - \frac{r^2}{a^2}B_z = 0$$

Let $\frac{r}{a} = x$, $\frac{dB_z}{dx} = \frac{dB_z}{dr}\cdot\frac{dr}{dx} = a\frac{dB_z}{dr}$, $\frac{d^2B_z}{dx^2} = a^2\frac{d^2B_z}{dr^2}$, then the equation will become

$$x^2\frac{d^2B_z}{dx^2} + x\frac{dB_z}{dx} - x^2B_z = 0$$

This equation is the modified Bessel's differential equation (at $\alpha = 0$)

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - (x^2 + \alpha^2)y = 0$$

having particular solutions $I_\alpha(x)$ and $K_\alpha(x)$ which are the modified Bessel's functions (the Bessel's functions of a purely imaginary argument) of the first and second kind respectively. In our case $\alpha = 0$, then particular solutions of the equation are $I_0(x)$ and $K_0(x)$.

The general solution is

$$B_z = CI_0(x) + DK_0(x),$$

where *C* and *D* are real constants

Replacing *x* with $\frac{r}{a}$ we get

$$B_z = CI_0\left(\frac{r}{a}\right) + DK_0\left(\frac{r}{a}\right)$$

Answer: $B_z = CI_0\left(\frac{r}{a}\right) + DK_0\left(\frac{r}{a}\right)$.