Answer on Question #62809 – Math – Differential Equations

Question

dB/dx=B/t2

t is constant. if we solve this we get B=B0 ex/t+B0e-x/t if the graph is DECAYING EXPONENTIALY then can we ignore the B0 ex/t term?

Solution

$B = B_0 e^{x/t} + B_0 e^{-x/t}$ is not a solution of equation

$$\frac{dB}{dx} = \frac{B}{t^2}$$

Because

$$\frac{d}{dx}\left(B_0 e^{\frac{x}{t}} + B_0 e^{-\frac{x}{t}}\right) = B_0\left(\frac{1}{t}e^{\frac{x}{t}} - \frac{1}{t}e^{-\frac{x}{t}}\right) = B_0 \frac{1}{t}\left(e^{\frac{x}{t}} - e^{-\frac{x}{t}}\right) \neq \frac{B}{t^2} = \frac{(B_0 e^{x/t} + B_0 e^{-x/t})}{t^2}$$

But, $B = B_0 e^{x/t} + B_0 e^{-x/t}$ is the solution of the equation

$$\frac{d^2B}{dx^2} = \frac{B}{t^2}$$

Because

$$\frac{d^2B}{dx^2} = \frac{d}{dx}\left(B_0 \frac{1}{t}\left(e^{\frac{x}{t}} - e^{-\frac{x}{t}}\right)\right) = B_0 \frac{1}{t}\frac{d}{dx}\left(e^{\frac{x}{t}} - e^{-\frac{x}{t}}\right) = B_0 \frac{1}{t}\left(\frac{1}{t}e^{\frac{x}{t}} + \frac{1}{t}e^{-\frac{x}{t}}\right) =$$

$$= B_0 \frac{1}{t^2}\left(e^{\frac{x}{t}} + e^{-\frac{x}{t}}\right) = \frac{B}{t^2}$$

Let's solve equation

$$\frac{d^2B}{dx^2} = \frac{B}{t^2}$$

We will find the solution in the form

$$B = e^{\lambda x}$$

Substituting into the equation yields

$$\frac{d^2B}{dx^2} = \lambda^2 e^{\lambda x} = \frac{1}{t^2}e^{\lambda x}$$

From this we obtain the characteristic equation: $\lambda^2 = \frac{1}{t^2}$. The solutions of this equation are

$\lambda = \pm\frac{1}{t}$.

Thus we have two particular solutions $B_1 = e^{x/t}$ and $B_2 = e^{-x/t}$.

The general solution of the original equation is the linear combination of the particular solutions:

$$B = Ce^{x/t} + De^{-x/t}$$

where C and D are some constants which we chose depending on initial or boundaries conditions, if $B \to 0$ when $x \to \infty$, we have to take $C = 0$, so we have $B = De^{-x/t}$ or $B = B_0 e^{-x/t}$ where $B = B_0$ at $x = 0$

Answer: $B = B_0 e^{-x/t}$.