Answer on Question #62780 – Math – Differential Equations

Question

Solve

$d^2Bz(x)/dx^2=Bz(x)/$ some contant^2
The z is in subscript and solve this equation.

Solution

Let a^2 be a constant. The solution of equation

$$\frac{d^2B_z(x)}{dx^2} = \frac{B_z(x)}{a^2}, \quad (1)$$

can be found in the following form:

$$B_z(x) = e^{\lambda x}. \quad (2)$$

Substituting (2) into the equation (1) obtain

$$\frac{d^2B_z(x)}{dx^2} = \lambda^2 e^{\lambda x} = \frac{1}{a^2} e^{\lambda x}. \quad (3)$$

From (3) we obtain the characteristic equation:

$$\lambda^2 = \frac{1}{a^2}. \quad (4)$$

The solutions of equation (4) are

$$\lambda = \frac{1}{a} \text{ and } \lambda = -\frac{1}{a}.$$

Then we obtain two particular solutions, namely

$$B_{z1}(x) = e^{x/a} \text{ and } B_{z2}(x) = e^{-x/a}.$$

The general solution of the original equation (1) is a linear combination of particular solutions:

$$B_z(x) = Ce^{x/a} + De^{-x/a},$$

where C and D are real constants.

The general solution of (1) can also be written as

$$B_z(x) = E\sinh(x/a) + F\cosh(x/a),$$

where E and F are real constants;

$\sinh(x/a) = \frac{1}{2}\left(e^{x/a} - e^{-x/a}\right)$ is the hyperbolic sine and $\cosh(x/a) = \frac{1}{2}\left(e^{x/a} + e^{-x/a}\right)$ is the hyperbolic cosine. These functions are particular solutions too.

Answer: $B_z(x) = Ce^{x/a} + De^{-x/a}$ or
$B_z(x) = E\sinh(x/a) + F\cosh(x/a)$.