Answer on Question #62636 – Math – Algorithms | Quantitative Methods

Question

Let f(n) and g(n) be functions with domain {1, 2, 3, . . .}.

Prove the following:
If $f(n) = \Omega(g(n))$, then $g(n) = O(f(n))$.

Solution

If $f(n) = \Omega(g(n))$, then, by definition of Ω, there exist positive constants c and n_0 such that

$$c|g(n)| \leq |f(n)| \text{ for all } n \geq n_0.$$

Hence,

$$|g(n)| \leq \frac{|f(n)|}{c}.$$

Set

$$\frac{1}{c} = k.$$

If c > 0, then k > 0.

Besides,

$$|g(n)| \leq k|f(n)|.$$

Therefore, there exist positive constants **k** and n_0 such that $|g(n)| \leq k|f(n)|$ for all $n \geq n_0$.

By definition of O,

$$g(n) = O(f(n)).$$