Answer on Question #62565 – Math – Differential Equations

Question

If $y = e^{ax} \cdot \cos^3 x \ \cdot \sin^2 x$, find $\frac{dy}{dx}$

Solution

$$\frac{dy}{dx} = \frac{d}{dx}(e^{ax} \cdot \cos^3 x \ \cdot \sin^2 x) = \frac{de^{ax}}{dx}\cos^3 x \sin^2 x + e^{ax}\frac{d\cos^3 x}{dx}\sin^2 x + e^{ax}\cos^3 x\frac{d\sin^2 x}{dx}$$

$$= e^{ax}(ax)' \cos^3 x \sin^2 x + e^{ax}\, 3\cos^2 x(\cos x)' \sin^2 x + e^{ax}\cos^3 x\, 2\sin x\,(\sin x)'$$

$$= ae^{ax}\cos^3 x \sin^2 x - 3e^{ax}\cos^2 x \sin^3 x + 2e^{ax}\cos^4 x \sin x$$

$$= e^{ax}\cos^2 x \sin x\,(a\cos x \sin x - 3\sin^2 x + 2\cos^2 x)$$

$$= \frac{1}{2}e^{ax}\cos x \sin 2x\left(\frac{a}{2}\sin 2x - \frac{3}{2}(1-\cos 2x) + (1+\cos 2x)\right) =$$

$$= \frac{1}{4}e^{ax}\cos x \sin 2x\,(a\sin 2x + 5\cos 2x - 1)$$

Answer: $\frac{dy}{dx} = \frac{1}{4}e^{ax}\cos x \sin 2x\,(a\sin 2x + 5\cos 2x - 1)$.