Answer on Question #62351 – Math – Combinatorics | Number Theory

Question

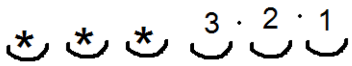
A panel is conducting an interview on six candidates of different heights. If they are to put them in line, in how many ways can they arrange them in line such that no three consecutive candidates are in increasing order of height from front to back?

Solution

Assign 6 different heights to 6 different numbers. We have 1 2 3 4 5 6. This is an increasing sequence.
First questions: in how many ways can we choose an increasing sequence of 3 numbers (123, 246, 456,…).

$$C_6^3 = \frac{6!}{3!\,3!} = 20$$

There are 6 places for numbers. If the sequence of three numbers is on the first place the remaining number you can select in 6 ways.



We can supply three numbers to other places, too.

There are 4 ways.
So, 20*6*4=480.
In this case, we have considered some numbers twice. Using rule of sum we have to fix it.
Second questions: in how many ways can we choose an increasing sequence of 4 numbers (1235, 2456, 1456, …).

$$C_6^4 = \frac{6!}{4!\,2!} = 15$$

There are 6 places for numbers. If the sequence of four numbers is on the first place the remaining number you can select in 2 ways.
We can supply four numbers to other places, too. There are 3 ways.
So, 15*2*3=90.
Third questions: in how many ways can we choose an increasing sequence of 5 numbers:

$$C_6^5 = \frac{6!}{5!\,1!} = 6$$

There are 6 places for numbers. If the sequence of five numbers is on the first place the remaining number you can select 1 way.
We can supply five numbers to other places, too. There are 2 ways.
So, 6*1*2=12.
And last sequence 6 5 4 3 2 1.
There are 6!=720 ways to put six candidates in line.
By the rule of sum 720 – 90 + 12 – 1 = 319 ways.
Answer: 319 ways.