## Answer on Question \#62254 - Math - Statistics and Probability

## Question

Let $A$ and $B$ be any two events defined on the same sample space. Suppose $P(A)=0.3$ and $P(A \cap B)=0.6$. Find $P(B)$ such that $A$ and $B$ are independent.

## Solution

Definition: Two events $A$ and $B$ are independent if $P(A \cap B)=P(A) P(B)$. Equivalently: $\mathrm{P}(\mathrm{A} \mid \mathrm{B})=\mathrm{P}(\mathrm{A}), \mathrm{P}(\mathrm{B} \mid \mathrm{A})=\mathrm{P}(\mathrm{B})$.
Therefore
$\mathrm{P}(\mathrm{B})=\mathrm{P}(\mathrm{A} \cap \mathrm{B}) / \mathrm{P}(\mathrm{A})=0.6 / 0.3=2$, which is false, because $2>1$.
Thus, the problem has no solution.
Answer: the problem has no solution.

## Question

Let A and B be any two events defined on the same sample space. Suppose $P(A)=0.3$ and $P(A \cup B)=0.6$. Find $P(B)$ such that $A$ and $B$ are independent.

## Solution

$\mathrm{P}(\mathrm{A} U \mathrm{~B})=\mathrm{P}(\mathrm{A})+\mathrm{P}(\mathrm{B})-\mathrm{P}(\mathrm{A} \cap \mathrm{B})$
If two events $A$ and $B$ are independent, then $P(A \cap B)=P(A) P(B)$.
Therefore
$\mathrm{P}(\mathrm{A} U \mathrm{~B})=\mathrm{P}(\mathrm{A})+\mathrm{P}(\mathrm{B})-\mathrm{P}(\mathrm{A}) \mathrm{P}(\mathrm{B})$;
$0.6=0.3+\mathrm{P}(\mathrm{B})-0.3 \mathrm{P}(\mathrm{B})$;
$0.7 \mathrm{P}(\mathrm{B})=0.3$;
$\mathrm{P}(\mathrm{B})=0.3 / 0.7=3 / 7$.
Answer: $\mathrm{P}(\mathrm{B})=3 / 7$.

