Answer on Question #62211 – Math – Statistics and Probability

Question

An unbiased coin is tossed three times. If A is the event that a head appears on each of the first two tosses, B is the event that a tail occurs on the third toss and C is the event that exactly two tails appear in the three tosses, show that:

i) Events A and B are independent;

ii) Events B and C are dependent.

Solution

Obviously $P(A) = P\{head, head,*\} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4},\ P(B) = P\{*,*, tail\} = \frac{1}{2}$, where * may be either head or tail.

To calculate $P(C) = P\{exactly\ two\ tails\ appear\ in\ the\ three\ tosses\}$ we must use the binomial distribution. In our case $n = 3$, $p = \frac{1}{2}$ (the probability of having the tail on one toss),

$q = 1 - p = \frac{1}{2}$. So $P(C) = C_3^2 \cdot \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^{3-2} = \frac{3!}{2!\cdot 1!} \cdot \left(\frac{1}{2}\right)^3 = \frac{3}{8}$.

i)

$P(A \cap B) = P\{head, head, tail\} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$.

Since $\frac{1}{8} = P(A \cap B) = P(A)P(B) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$, we conclude that events A and B are independent.

ii)

$P(B \cap C) = P\{tail, head, tail\} + P\{head, tail, tail\} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.

Since $\frac{1}{4} = P(B \cap C) \neq P(B)P(C) = \frac{1}{2} \cdot \frac{3}{8} = \frac{3}{16}$, we conclude that events B and C are dependent.