

Answer on Question #62103 – Math – Statistics and Probability

Question

3. Each of 12 refrigerators of a certain type has been returned to a distributor because of an audible, high-pitched, oscillating noise when the refrigerator is running. Suppose that 7 of these refrigerators have defective compressor and the other 5 have less serious problems. If the refrigerators are examined in random order. Let X be the number among the first 6 examined that have a defective compressor. What is the probability that X exceeds its mean value by more than 1 standard deviation?

Solution

It is hypergeometric probability distribution with $N = 12, n = 6, M = 7, N - M = 5$.

$$\mu = n \frac{M}{N} = 6 \frac{7}{12} = 3.5;$$

$$\sigma = \sqrt{n \frac{M}{N} \frac{N-M}{N} \frac{N-n}{N-1}} = \sqrt{6 \frac{7}{12} \frac{5}{12} \frac{12-6}{12-1}} = 0.89;$$

$$\mu + \sigma = 3.5 + 0.89 = 4.39.$$

The probability that X exceeds its mean value by more than 1 standard deviation is

$$P(X > \mu + \sigma) = 1 - P(X < \mu + \sigma) = 1 - F(\mu + \sigma).$$

We used Excel function HYPGEOM.DIST:

$$F(4.39) = \text{HYPGEOM.DIST}(4.39, 6, 7, 12, \text{TRUE}) = 0.8788$$

$$P(X > \mu + \sigma) = 1 - 0.8788 = 0.1212.$$

Answer: 0.1212.

Question

4. The error involved in making a certain measurement is a continuous rv X with pdf

$$f(x) = \begin{cases} 0.09375(4 - x^2), & -2 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

Compute $P(-1 < x < 1)$.

Solution

$$P(-1 < x < 1) = \int_{-1}^1 0.09375(4 - x^2) = 0.09375 \left(4x - \frac{x^3}{3} \right)_{-1}^1 = 0.09375 \left(8 - \frac{2}{3} \right) = 0.6875.$$

Answer: 0.6875.