**Answer on Question #62102 – Math – Statistics and Probability**

## Question

1. A certain shop repairs both audio and video components.
Let A denote the event that the next component brought in for repair is an audio component, and let B be the event that the next component is a compact disc player (so the event B is contained in A). Suppose that P(A)= 0.6 and P(B) =0.05. What is P(B|A)?
0.027
0.064
0.072
0.083

## Solution

Since B is contained in A, then $A \cap B = B$ and

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(B)}{P(A)} = \frac{0.05}{0.6} = \frac{1}{12} \approx 0.083.$$

**Answer:** $P(B|A) = \frac{1}{12} \approx 0.083.$

## Question

2. Components of a certain type are shipped to a supplier in batches of ten. Suppose that 50% of all such batches contain no defective components, 30% contain one defective component, and 20% contain two defective components. Two components from a batch are randomly selected and tested. What are the probabilities associated with 0, 1, and 2 defective components being in the batch under the condition that neither tested component is defective.
0.677
0.512
0.578
0.617

## Solution

Let $B_0$ be the event that batch has 0 defectives, $B_1$ be the event that batch has 1 defective, and $B_2$ be the event that batch has 2 defectives. Let $C$ be the event that neither selected component is defective.
It is given that

$$P(B_0) = 0.5, P(B_1) = 0.3, P(B_2) = 0.2.$$

The event $C$ can happen in three different ways:

(i) The batch of 10 is perfect, and we get no defectives in the sample of two:

$P(C|B_0) = 1;$

(ii) The batch of 10 has 1 defective, but the sample of two misses them:

$$P(C|B_1) = \frac{\binom{9}{2}}{\binom{10}{2}} = \frac{9!\cdot 2!\cdot(10-2)!}{2!\cdot(9-2)!\cdot 10!} = \frac{8}{10};$$

(iii) The batch has 2 defective, but the sample misses them:

$$P(C|B_2) = \frac{\binom{8}{2}}{\binom{10}{2}} = \frac{8!\cdot 2!\cdot(10-2)!}{2!\cdot(8-2)!\cdot 10!} = \frac{56}{90}.$$

According to the total probability formula,

$$P(C) = P(C|B_0)P(B_0) + P(C|B_1)P(B_1) + P(C|B_2)P(B_2) =$$
$$= 1 \cdot 0.5 + \frac{8}{10} \cdot 0.3 + \frac{56}{90} \cdot 0.2 = \frac{389}{450}.$$

Using Bayes' law

$$P(B_0|C) = \frac{P(C|B_0)P(B_0)}{P(C)} = \frac{1\cdot 0.5}{\frac{389}{450}} = \frac{225}{389} = 0.5784,$$

$$P(B_1|C) = \frac{P(C|B_1)P(B_1)}{P(C)} = \frac{\frac{8}{10}\cdot 0.3}{\frac{389}{450}} = \frac{108}{389} = 0.2776,$$

$$P(B_2|C) = \frac{P(C|B_2)P(B_2)}{P(C)} = \frac{\frac{56}{90}\cdot 0.2}{\frac{389}{450}} = \frac{56}{389} = 0.1440.$$

**Answer:** $0.5784$; $0.2776$; $0.1440$.